MATH 481 – Introduction to Stochastic Processes

Course Description from Bulletin: This is an introductory course in stochastic processes. Its purpose is to introduce students into a range of stochastic processes, which are used as modeling tools in diverse fields of applications, especially in the business applications. The course introduces the most fundamental ideas in the area of modeling and analysis of real World phenomena in terms of stochastic processes. The course covers different classes of Markov processes: discrete and continuous-time Markov chains, Brownian motion and diffusion processes. It also presents some aspects of stochastic calculus with emphasis on the application to financial modeling and financial engineering. Credit may not be granted for MATH 481 and MATH 542. (3-0-3).

Enrollment: Elective for AM and other majors

Textbook(s): W. Gregory F. Lawler, *Introduction to Stochastic Processes*, Chapman & Hall;
Thomas Mikosch, *Elementary Stochastic Calculus with Finance in View*, World

Other required material: None

Prerequisites: MATH 332 or 333 or equivalent; MATH 475

Objectives:

1. Students will understand the basic concepts underlying the theory and practice of finite and countably infinite Markov chains in discrete time and in continuous time.
2. Students will understand basic concepts underlying the theory and practice martingales in discrete time and in continuous time.
3. Students will understand some aspects of the elementary stochastic analysis (stochastic integral, Ito formula and Girsanov theorem -- all for Brownian motion).

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:	Hours
1. Discrete-time Markov chains	9
a. Motivation and construction	
b. First step analysis and Chapman-Kolmogorov equations	
c. Long-range behavior and invariant probability	
d. Classification of states	
e. Return times [first return times, mean return times]	
2. Discrete-time martingales	12
a. Filtrations and conditional expectations	
b. Definitions and examples	
c. Stopping times, Markov times, optional sampling theorem and optional stopping theorem	
d. Uniform integrability and UI martingales	
e. Martingale convergence theorem	

    f. Doob-Meyer decomposition
    g. The quadratic variation process
3. Continuous-time Markov chains 3
    a. Poisson process
    b. Birth and Death process
4. Brownian motion process 6
    a. Definition and basic properties
    b. Markov property
    c. Functionals of Brownian motion
    d. Brownian motion with a drift and geometric Brownian motion
5. Continuous-time Markov processes and martingales 6
    a. Definition and examples
    b. Diffusion process
6. Elements of stochastic analysis 9
    a. Stochastic integration
    b. Ito formula and (Stochastic) Integration by parts formula
    c. Stochastic differential equations, diffusion processes, Ito processes
    d. Girsanov transformation

Assessment:	Homework	0-10%
	Quizzes/Tests	45-50%
	Final Exam	45-50%

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Date: 03/11/06