MATH 553 – Discrete Applied Mathematics I

Course Description from Bulletin: Graph theory is the study of systems of points with some of the pairs of points joined by lines. Sample topics include: paths, cycles and trees; adjacency and connectivity; directed graphs; Hamiltonian and Eulerian graphs and digraphs; intersection graphs. Applications to the sciences (computer, life, physical, social) and engineering will be introduced throughout the course. This course run concurrently with MATH 454 but projects and homework are at the graduate level. Credit will not be granted for both MATH 454 and MATH 553. (3-0-3)

Enrollment: Graduate Core Course

Textbook(s): West, *Introduction to Graph Theory,* 2nd ed., Prentice Hall

Other required material:

Prerequisites: MATH 453 or instructor's consent.

Objectives:
1. Students will achieve command of the fundamental definitions and concepts of graph theory.
2. Students will understand and apply the core theorems and algorithms, generating examples as needed, and asking the next natural question.
3. Students will achieve proficiency in writing proofs, including those using basic graph theory proof techniques such as bijections, minimal counterexamples, and loaded induction.
4. Students will become familiar with the major viewpoints and goals of graph theory: classification, extremality, optimization and sharpness, algorithms, and duality.
5. Students will be able to apply their knowledge of graph theory to problems in other areas, possibly demonstrated by a class project.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:	Hours
1. Review of induction and proof techniques	2
2. Fundamental Concepts	7
a. Basic definitions of graphs and multigraphs; adjacency matrices, isomorphism, girth, decompositions, independent sets and cliques, graph complements, vertex coloring, chromatic number, and the Petersen graph	
b. Paths, cycles, and trails; Eulerian circuits	
c. Vertex degrees and counting; large bipartite subgraphs, the handshaking lemma	
d. Directed graphs: weak connectivity, connectivity, strong components	
3. Trees and Distance	7

a. Basic properties: equivalent characterizations, edge exchanges, forests
b. Spanning trees and enumeration: deletion-contraction, Cayley's Theorem, Matrix-Tree Theorem
c. Optimization and trees: minimum spanning tree, Kruskal's algorithm and variations, Dijkstra's algorithm

4. Matchings and Factors 7
   a. Matchings and covers: bipartite matching, vertex covers, edge covers, M-alternating paths, Hall's Theorem, Konig-Egevary Theorem
   b. Algorithms and applications: Hungarian algorithm
   c. Matchings in general graphs: Tutte's Theorem
5. Connectivity and Paths 6
   a. Cuts and connectivity: vertex and edge connectivity, vertex cuts and separating sets, Harary graphs, block-cutpoint graph and depth-first search
   b. *k*-connected graphs: (closed) ear decomposition, Menger's Theorem(s), line graphs
   c. Network flow problems: Max-flow/Min-cut Theorem and Ford-Fulkerson Labeling algorithm, flow integrality, proof of Menger's Theorem

Coloring of Graphs

6. Vertex Coloring 5
   a. Chromatic Number: lower bounds from clique number and maximum independent set, upper bounds from greedy coloring (& Welsh-Powell), Szekeres-Wilf, and Brooks' Theorem. Also k-critical graphs, cartesian product of graphs, and interval graphs.
   b. k-Chromatic graphs: Mycielski's construction, Turan's Theorem, color-critical graphs
7. Planarity 5
   a. Embeddings, dual graphs, Euler's formula
   b. Kuratowski's Theorem and graph minors
   c. Coloring
8. Edges and Cycles 2
   a. Edge coloring: bipartite graphs, Vizing's Theorem, Shannon's bound
   b. Hamiltonian Cycles, 2-approximation of Traveling Salesman Problem on planar graphs

Assessment:		
	Homework/Project	10-50%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Robert Ellis and Michael Pelsmajer
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