Math 554 - Discrete Applied Mathematics II

Course Description from Bulletin: Graduate level treatment of applied combinatorics; posets: product and dimension, lattices, extremal set theory and symmetric chain decomposition; combinatorial designs: block designs, Latin Squares, finite fields, block designs and Steiner systems, finite projective planes; coding theory: error-correcting codes, Hamming and sphere bounds, linear codes, codes from designs, liar games and adaptive coding. (3-0-3)

Enrollment: Graduate core course.

Textbook(s): Roberts and Tesman, *Applied Combinatorics*, 2nd Edition, Prentice Hall.

or

Peter J. Cameron, *Combinatorics: Topics, Techniques, Algorithms*, Cambridge University Press, 1994.

Other required material:

Prerequisites: MATH 453, MATH 454 or MATH 553

Objectives:

1. Students will be familiar with many core topics from posets, combinatorial designs, set systems, coding theory, and applications.
2. Students will learn how to approach topics from various points of view: structural (including duality), extremal, algorithmic, constructive.
3. Students will learn many proof techniques of discrete mathematics, especially variations of strong induction/minimal counterexample/extremal choice, double counting, and constructions using finite fields, combinatorial proofs via linear algebraic structure, as well as reviewing basic combinatorics like Stirling's Formula and combinatorial identities.
4. Students will do a project with a presentation on material approved by the instructor. Previous projects in similar classes have included applications of class topics to student's own research area, and expository talks (with proofs) on material not covered during class.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures

Course Outline: Hours

1. Partially Ordered Sets, Lattices, and Set Systems 13
    a. Dilworth's Theorem (& Mirsky's “dual”)
    b. Sperner's Theorem
    c. Symmetric chain decomposition & application to data allocation
    d. Erdos-Ko-Rado Theorem for an intersecting family
    e. Product and Dimension of Posets
    f. Distributive Lattices
    g. Arrow's Theorem (optional)

2. Design Theory 13
    a. Block Designs
    b. Latin Squares: Orthogonal Arrays and Families, with Applications to Cryptography
    c. Finite Fields and Families of Latin Squares: RSA Cryptosystem Complete Orthogonal Family of Latin Squares
    d. Balanced Incomplete Block Designs: Existence, Fisher's Inequality, Steiner Triple Systems, Applications
    e. Finite Projective Planes
3. Coding Theory 13
    a. Information Transmission, Encoding, and Decoding
    b. Error-Correcting Codes: Hamming Distance, Hamming and Sphere Bounds, Consensus Decoding and Molecular Sequences
    c. Linear Codes: Generator Matrices, Error-Correction, Hamming Codes
    d. Block Designs for Error-Correcting Codes: Hadamard Codes, Applications
    e. Hadamard matrices & Reed-Muller codes
    f. Liar Games: Adaptive Error-Correcting and Covering Codes
4. Student Presentations 3

Assessment:	Homework/Midterm Exam	30-60%
	Presentation	10-20%
	Final Exam	30-50%

Syllabus prepared by: Michael Pelsmajer and Robert Ellis
Date: 3/07/06