Math 431 – Applied Algebra II

Course Description from Bulletin: Ring homomorphisms; factorization and reducibility in polynomial rings; integral domains; vector spaces; fields and their extensions. As time permits, applications to one or more of the following: Frieze and crystallographic groups, Caley digraphs, and coding theory. (3-0-3)

Enrollment: Elective for AM and other majors.

Textbook(s): Gallian, *Contemporary Abstract Algebra, 6th* Edition, Houghton Mifflin.
Supplementary text (available through Galvin Library): Huffman and Pless, *Fundamentals of Error Correcting-Codes*, 1st Edition, Cambridge University Press, 2003.

Other required material: None

Prerequisites: MATH 430

Objectives:

1. Students will achieve command of the fundamental definitions and concepts of rings and fields; and apply the theory of groups, rings, and fields to applications selected from one or more of algebraic coding theory, Cayley digraphs, Frieze and crystallographic groups, and Burnside's Theorem for counting.
2. Students will understand and apply the core theorems and algorithms, generating examples as needed, and asking the next natural question.
3. Students will achieve proficiency in writing proofs, including those using basic number theory, induction, homomorphisms and isomorphisms.
4. Students will become familiar with the major viewpoints and goals of abstract algebra: classification, symmetry, algorithms and structure.
5. Students will practice their knowledge of abstract algebra to problems with exercises and applications, possibly through the use of a computer algebra system or a class project.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:	Hours
1. Polynomial Rings	2
2. Factorization of Polynomials Reducibility tests, irreducibility tests, unique factorization	3
3. Divisibility in Integral Domains Irreducibles, primes, Fermat's last theorem, unique factorization, Euclidean domains	3
4. Vector Spaces	2
5. Extension Fields Splitting fields, zeroes of polynomials	3
6. Algebraic Extensions	2
7. Finite Fields	2

8. Geometric Constructions 2
9. Sylow Theorems 3
10. Generators and Relations 3
11. Frieze Groups and Crystallographic Groups 4
12. Cayley Digraphs of Groups 4
13. Introduction to Algebraic Coding Theory 4
    Linear codes, parity-check matrices, coset decoding
    Reed-Solomon Codes
14. Miscellaneous Topics in Coding Theory 5
    Constructions for linear codes; Hamming, Golay, and Reed-Muller codes; sphere packing bound, packing and covering radius, and perfect codes; liar games

Note: Some of the Sections 9-14 may be covered in less depth depending on time constraints. In some semesters, Sections 1-3 might be sufficiently covered in Math 430 (Applied Algebra), allowing more time for Sections 4-14.

Assessment:	Homework	10-30%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Robert Ellis and Michael Pelsmajer
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