Illinois Institute of Technology
Department of Biological, Chemical, and Physical Sciences

Physics Division

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Wednesday March 27, 2002

1:00 – 5:00 PM

General Instructions

1. Each problem is to be done in a separate blue book. Label the cover of each blue book with your identifying code letter, the part number of the exam, and the number of the problem only; for example: A-II.6. Do not write your name on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will solve every problem.

Physical Constants

Speed of light in vacuum	c	$=$	2.998×10^8 m/s
Planck's constant	h	$=$	6.626×10^{-34} J·s
	$\hbar$	$=$	$h/2\pi$
		$=$	1.055×10^{-34} J·s
		$=$	6.582×10^{-16} eV·s
Permeability constant	μ_o	$=$	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	$=$	8.988×10^9 N·m^2/C^2
Fine structure constant	α	$=$	$\frac{e^2}{4\pi\epsilon_o\hbar c}$
		$=$	$7.30 \times 10^{-3} \approx \frac{1}{137}$
Gravitational constant	G	$=$	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	$=$	6.023×10^{23} mol^{-1}
Boltzmann's constant	k	$=$	1.381×10^{-23} J/K
		$=$	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	$=$	0.0258 eV
Universal gas constant	R	$=$	8.314 J/mol·K = 1.99 cal/mol·K
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	$=$	1.602×10^{-19} C
Electron rest mass	m_e	$=$	9.109×10^{-31} kg
		$=$	0.5110 MeV/c^2
Neutron rest mass	m_n	$=$	1.675×10^{-27} kg
		$=$	939.6 MeV/c^2
Proton rest mass	m_p	$=$	1.672×10^{-27} kg
		$=$	938.3 MeV/c^2
Deuteron rest mass	m_d	$=$	3.343×10^{-27} kg
		$=$	1875.6 MeV/c^2
Atomic mass unit (Dalton) ($C^{12} = 12$)	Da	$=$	1.661×10^{-27} kg
		$=$	931.5 MeV/c^2
Mass of Earth	M_E	$=$	5.98×10^{24} kg
Radius of Earth	R_E	$=$	6.37×10^6 m
Mass of Sun	M_S	$=$	1.99×10^{30} kg
Radius of Sun	R_S	$=$	6.96×10^8 m
Gravitational acceleration at Earth's surface	g	$=$	9.81 m/s^2
Atmospheric pressure		$=$	1.01×10^5 N/m^2
Radius of Earth's orbit		$=$	1.50×10^{11} m
Radius of Moon's orbit		$=$	3.84×10^8 m

Conversion Factors

1 eV	$=$	1.602×10^{-19} J	1 J	$=$	6.242×10^{18} eV
1 Å	$=$	10^{-10} m	1 fermi	$=$	10^{-15} m
1 barn (b)	$=$	10^{-28} m^2	1 in	$=$	2.54 cm
0° Celsius	$=$	273.16 K	1 cal	$=$	4.19 J

1. Consider a monatomic ideal gas consisting of N atoms of mass m contained in a volume V. Starting with the partition function of the system,

$$Q = \sum_j e^{-\beta E_j} \ ,$$

where $\beta = 1/k_B T$ and the sum is over all energy states, find the following:

   (a) Helmholtz free energy

   (b) entropy

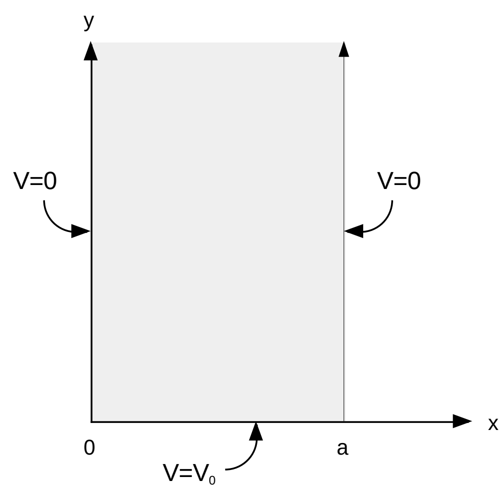
   (c) internal energy

2. Show that the chemical potential of a Fermi gas in two dimensions is given by

$$\mu(T) \ = \ k_B T \ \ln \left[\exp \left(\frac{\pi n \hbar^2}{m k_B T} \right) - 1 \right]$$

for n electrons (of mass m) per unit area. Note that the density of states of a free-electron gas in two dimensions is independent of energy: $N(E) \ = \ m/(\pi\hbar^2)$ per unit area of specimen.

3. Solve Laplace's Equation in the shaded region shown in the figure, *i.e.*, for $0 \leq x \leq a$, $0 \leq y < \infty$.



The boundary conditions are

$$\begin{array}{lll} V = 0 & \text{at} & x = 0 \text{ and } a \\ V = V_0 & \text{along} & y = 0 \end{array}$$

4. An infinite series can be summed using the formula

$$\sum_{n=-\infty}^{n=+\infty} F(n) \ = \ -\pi \sum_i r_i \cot(\pi z_i) \qquad (1)$$

where the function $F(z)$ has simple poles at z_i, each of which has residue r_i.

Use this formula to show that

$$\sum_{n=-\infty}^{n=+\infty} \frac{1}{n^4 + a^4} = \frac{\pi}{\sqrt{2}a^3} \frac{\sinh(\pi a\sqrt{2}) + \sin(\pi a\sqrt{2})}{\cosh(\pi a\sqrt{2}) - \cos(\pi a\sqrt{2})} . \tag{2}$$

5. Consider the following trial wavefunction for the symmetric linear potential well:

$$\psi_{\text{trial}}(x) = \frac{1}{\sqrt{a\pi}} \exp\left(-\frac{x^2}{2a^2}\right) .$$

Estimate the ground-state energy of the well with this wavefunction.

6. In the hydrogen atom, the effect of the finite size of the nucleus on the observable energy levels can be taken into account by modifying the Coulomb potential at short distances according to

$$V(r) = \begin{cases} -\frac{Ke^2}{2R^3}(3R^2 - r^2) & 0 < r < R \\ -\frac{Ke^2}{r} & r \geq R \end{cases}$$

Estimate the shift in the energy levels of the 1S and 2S states due to this modification of the usual Coulomb potential given by $V(r) = -Ke^2/r$ for $0 < r < \infty$.

7. Using the (relativistic) kinematics of the Compton-scattering process (photon on free electron of rest mass m):

   (a) In the lab frame, *i.e.*, the frame in which the electron is initially at rest, what is the change in wavelength of a photon scattered through an angle θ?

   (b) In the center-of-momentum frame, what is the change in wavelength of a photon scattered through an angle ϕ?

8. (a) There is no known meson with charge $Q = +1$ and strangeness $S = -1$, or with $Q = -1$ and $S = +1$. Explain why, in terms of the quark model.

   (b) The Ξ^- hyperon normally decays according to $\Xi^- \to \Lambda\pi^-$, with a measured branching ratio of 99.887%. In contrast, the experimental upper limit on the branching ratio for the $\Xi^- \to p\pi^-\pi^-$ decay, where the proton and pion do not form a Lambda, is 4×10^{-4} (at the 90% confidence level). What prevents the latter decay from occurring? Illustrate your argument with appropriate Feynman diagrams. (Recall that the quark content of the Ξ^- is ssd, and that of the Λ is uds.)