Illinois Institute of Technology

Department of Biological, Chemical, and Physical Sciences

Physics Division

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Monday March 24, 2003

1:00 – 5:00 PM

General Instructions

1. Each problem is to be done in a separate blue book. Label the cover of each blue book with your identifying code letter, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name on any material handed in for grading.
2. Any data not specified in a problem should be found in the table at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will solve every problem.

Physical Constants

Speed of light in vacuum	c	$=$	2.998×10^{8} m/s
Planck's constant	h	$=$	6.626×10^{-34} J·s
	$\hbar$	$=$	$h/2\pi$
		$=$	1.055×10^{-34} J·s $= 6.582 \times 10^{-16}$ eV·s
Permeability constant	μ_o	$=$	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	$=$	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	$=$	$\frac{e^2}{4\pi\epsilon_o\hbar c} = 7.30 \times 10^{-3} \approx \frac{1}{137}$
Gravitational constant	G	$=$	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	$=$	6.023×10^{23} mol^{-1}
Boltzmann's constant	k	$=$	1.381×10^{-23} J/K $= 8.617 \times 10^{-5}$ eV/K
kT at room temperature	k·300 K	$=$	0.0258 eV
Universal gas constant	R	$=$	8.314 J/mol·K = 1.99 cal/mol·K
Stefan-Boltzmann constant	σ	$=$	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	$=$	1.602×10^{-19} C
Electron rest mass	m_e	$=$	9.109×10^{-31} kg $= 0.5110$ MeV/c^2
Neutron rest mass	m_n	$=$	1.675×10^{-27} kg $= 939.6$ MeV/c^2
Proton rest mass	m_p	$=$	1.672×10^{-27} kg $= 938.3$ MeV/c^2
Deuteron rest mass	m_d	$=$	3.343×10^{-27} kg $= 1875.6$ MeV/c^2
Atomic mass unit ($C^{12} = 12$)	u	$=$	1.661×10^{-27} kg $= 931.5$ MeV/c^2
Mass of Earth	M_E	$=$	5.98×10^{24} kg
Radius of Earth	R_E	$=$	6.37×10^{6} m
Mass of Sun	M_S	$=$	1.99×10^{30} kg
Radius of Sun	R_S	$=$	6.96×10^{8} m
Gravitational acceleration at Earth's surface	g	$=$	9.81 m/s^2
Atmospheric pressure		$=$	1.01×10^{5} N/m^2
Radius of Earth's orbit		$=$	1.50×10^{11} m
Radius of Moon's orbit		$=$	3.84×10^{8} m

Special Functions

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) \qquad Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi} \qquad Y_l^{-m} = (-1)^m (Y_l^m)^*$$

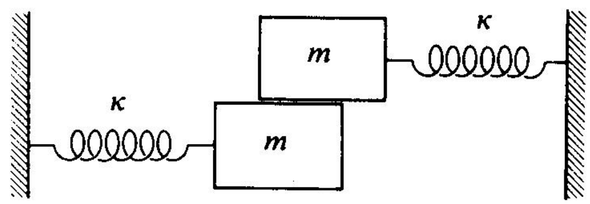
Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 Ci	=	3.7×10^{10} disintegrations/s
0° Celsius	=	273.16 K	1 cal	=	4.19 J

1. A satellite travels in a circular orbit of radius r_0. Its rocket motor fires, suddenly increasing its velocity by 8% along its direction of motion. What is the apogee of the new orbit? Make a sketch superimposing the new orbit on the original orbit.

2. Two identical harmonic oscillators are placed such that the two masses slide against one another, as in Fig. 1. The frictional force provides a coupling of the motions proportional to the instantaneous relative velocity. Discuss the coupled oscillations of the system: solve the equations of motion and characterize the possible modes of oscillation.

Figure 1:



3. Given a vector $V = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and a tensor $T = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, both referred to a 2-dimensional Cartesian coordinate system (x_1, x_2), compute the components of the vector and tensor referred to the new coordinate system obtained by a rotation of the old system by 90° in a positive sense about the $x_1 \times x_2$ axis.

4. An object is placed 1.0 m in front of a converging lens, of focal length 0.50 m, which is 2.0 m in front of a plane mirror.

   a) Where is the final image, as measured from the lens, that would be seen by an eye looking toward the mirror through the lens (and just past the object)?

   b) Is the final image real or virtual?

   c) Is the orientation of the final image the same as the object or inverted?

   d) What is the lateral magnification?

5. In a joint Soviet–French experiment to monitor the Moon's surface with a light beam, pulsed radiation from a ruby laser ($\lambda = 0.69\,\mu$m) was directed at the Moon through a reflecting telescope with a mirror radius of 1.3 m. A reflector on the Moon behaved like a circular plane mirror with radius 10 cm, reflecting the light directly back toward the telescope on Earth. The reflected light was then detected after being brought to a focus by this telescope. Assuming that for each direction of travel all the energy is in the central diffraction peak, what fraction of the original light energy was picked up by the detector?

6. Suppose that a deep shaft were drilled in Earth's crust near one of the poles, where the surface temperature is -40°C, to a depth where the temperature is 800°C.

a) What is the theoretical limit to the efficiency of an engine operating between these temperatures?

b) If all the energy released as heat into the low-temperature reservoir were used to melt ice that was initially at $-40°\text{C}$, at what rate could liquid water at $0°\text{C}$ be produced by such a power plant (which you should treat as an engine) of 100-MW capacity?

The specific heat of ice is $2220\,\text{J/kg·K}$; water's heat of fusion is $333\,\text{kJ/kg}$.

7. Show that in a lightly damped RLC circuit ($R \ll \omega L$), the fraction of the energy lost per oscillation is given to good approximation by $2\pi R/\omega L$.

8. A spherical shell is uniformly charged and undergoes purely radial oscillations. Show that this type of oscillating charge distribution cannot generate electromagnetic radiation. (Assume that the observer is far from the shell.)