Multicast Throughput of Hybrid Wireless Networks Under Gaussian Channel Model

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***Abstract*—We study the multicast capacity for hybrid wireless networks consisting of ordinary wireless nodes and base stations under *Gaussian Channel* model, which generalizes both the unicast capacity and broadcast capacity for hybrid wireless networks. We simply consider the *hybrid extended network*, where the ordinary wireless nodes are placed in the square region $\mathcal{A}(n)$ with side-length $\sqrt{n}$ according to a Poisson point process with unit intensity. In addition, m additional base stations (BSs) serving as the relaying communication gateway are placed regularly in the region $\mathcal{A}(n)$ and they are connected by a high-bandwidth wired network. Assume that all ordinary nodes, and the BSs when they deliver data to ordinary nodes, transmit at constant power P, and the power decays along path, with attenuation exponent $\alpha > 2$. The data rate of a transmission is determined by the SINR as $B\log(1+\text{SINR})$. There are n_s ordinary nodes to be randomly and independently chosen as the source of multicast sessions. Each source has n_d randomly chosen terminals.**

Three broad categories of multicast strategies are proposed. The first is the *hybrid strategy*, *i.e.*, the multihop scheme with BS-supported, which further consists of two types of schemes called *connectivity strategy* and *percolation strategy* respectively. The second is the *ordinary ad hoc strategy*, *i.e.*, the multihop scheme without any BS-supported. The third is the classical BS-based network protocol, *i.e.*, any communications between ordinary nodes are relayed by some specific BSs. According to the different scenarios in terms of m, n and n_d, we select the optimal scheme from the three categories of strategies, and derive the achievable multicast throughput based on the optimal decision. We show that the optimal decision can be classified into three cases in terms of m: When m: $[1, n/(\log n)^{\alpha+1}]$, if $n_d : [1, n/(\log n)^{3\alpha+2}]$ and $m : [\sqrt{nn_d \cdot (\log n)^{\alpha}}, n/(\log n)^{\alpha+1}]$, we should adopt the *hybrid strategy*; otherwise, we should adopt the *ordinary ad hoc strategy*. When m: $[n/(\log n)^{\alpha+1}, n/\log n]$, if $n_d : [1, n/(\log n)^{\alpha+2}]$ and $m : [\sqrt{nn_d \cdot (\log n)^{\alpha}}, n/\log n]$, we should adopt the *hybrid strategy*; otherwise, we should adopt the *ordinary ad hoc strategy*. When m: $[n/\log n, n]$, the *BS-based strategy* should be adopted.

***Index Terms*—wireless hybrid networks, wireless ad hoc networks, multicast throughput, random networks, multicast capacity, Gaussian channel model**

I. Introduction

The asymptotic capacity for wireless ad hoc networks has been intensively studied under difference channel models. Most existing related works are based on two types of channel models. The first is the *threshold-based channel* model that define the transmission rate as a binary function. The *protocol interference* model (PrIM) and *physical interference* model (PhIM) [5] both belong to the *threshold-based channel* model. The second is the *Gaussian Channel* model that determines the transmission rate based on a continuous function of the receiver's SINR (Signal to Interference plus Noise Ratio). *Gaussian Channel* model captures better the physical layer of wireless networks than *threshold-based channel* model that is a very crude approximation for wireless networks, under which any communication pair v_i and v_j can establish a direct communication link, over a channel of bandwidth B, of rate $R(v_i, v_j) = B\log(1 + \text{SINR}(v_j))$, *i.e.*, the receiver achieves the Shannon's capacity for a wireless channel with additive Gaussian white noise, see [2], [18].

A *hybrid wireless network* (HN) consists of two types of network terminals: base stations and ordinary wireless nodes. Assume that all base stations can communicate with wireless nodes, and further assume that each base station is neither a source nor a receiver, it simply serves as a relaying gateway. Intuitively, wireless ad hoc networks and cellular networks can both be regarded as the specific cases of the HN, as the number of base stations is adjusted. Thus, the study on capacity for HN has more generality than that for the wireless ad hoc networks and cellular network, while it was relatively not fully studied. In addition, as we know, multicast capacity can unify the unicast and broadcast capacity, [12], [13], which increases the generality of the research on multicast capacity for HN. For HNs, there are also generally two channel models as in most existing works for wireless ad hoc networks. Since all existing results of capacity for hybrid networks are derived under the *threshold-based* model, [16], a natural and interesting issue arises: What is the multicast capacity for hybrid networks when the Gaussian channel model is used. This paper aims to derive an achievable multicast capacity for HN under Gaussian channel model.

We consider *hybrid extended network* in which the ordinary wireless nodes are placed in the square region $\mathcal{A}(n)$ with side-length $\sqrt{n}$ according to a Poisson point process with unit intensity, and m additional base stations (BSs) serving as the relaying communication gateway are placed regularly in the region $\mathcal{A}(n)$, furthermore, we assume all base stations are connected by a high-bandwidth wired network. According to different cases in terms of m, n and n_d, we adopt different types of multicast strategies. To be specific, we propose three broad categories of multicast strategies. The first is called the *hybrid strategy*, *i.e.*, the multihop scheme with BS-supported, which further consists of two types of schemes called *connectivity strategy* and *percolation strategy* respectively. The second is the *ordinary ad hoc strategy*, *i.e.*, the multihop scheme without any BS-supported. The third is the classical BS-based network protocol, *i.e.*, any communications between ordinary nodes are relayed by some specific BSs. According to the different scenarios of m, n and n_d, we select the optimal scheme from the three categories of strategies, and derive the achievable multicast throughput based on the optimal scheme.

To the best of our knowledge, this is the first work that address the multicast routing and scheduling strategy in hybrid wireless networks under Gaussian channel model. The rest paper is structured as follows. In Section II, we introduce the network model. Main results are presented and discussed in Section III. We make technical preparations in Section IV. In Section V, we design the multicast schemes for HEN. In Section VI, we review the related existing literature. In Section VII, we conclude the paper.

II. Network Model

Throughout this paper, we denote the probability of an event E as $\Pr(E)$, and we are mainly concerned with events that take place with high probability (w.h.p.), *i.e.*, with probability 1 as the number of nodes $n \to \infty$.

A. Network topology

We construct a *random extended network* by placing ordinary nodes according to a Poisson point process (p.p.p.) of unit intensity on the 2-dimension plane and focusing on the square $\mathcal{A}(n)) = [0, \sqrt{n}]^2$. By Chebyshev's Inequality (Lemma 1), we easily obtain the number of nodes in $\mathcal{A}(n)$ is within $((1-\varepsilon)n, (1+\varepsilon)n)$. To simplify the description, we assume that the number of nodes are n, without changing our results in order sense, [4], [21]. Furthermore, we place regularly a number of base stations (with wireless transmission power P) in the region $\mathcal{A}(n)$, and they are connected using a high-bandwidth wired network, to construct the *hybrid extended network*. We assume that the number of BSs $m = O(n)$. Without loss of compatibility to most existing works, we assume $n_s = \Theta(n)$.

B. Achievable multicast throughput

The achievable multicast throughput is indeed a lower bound of the multicast capacity. In this paper, we follow the formal definitions of capacity in [12], [13], and we focus on the *minimum per-session multicast capacity*. Other two types of capacity, *i.e.*, *average per-session multicast capacity* and *aggregated multicast capacity*, can be straightforwardly derived based on *minimum per-session multicast capacity*. (See details in [12], [13]).

C. Channel model

Assume that all nodes transmit with a constant power P, and any two nodes can establish a direct communication link over a channel of bandwidth B, of rate $R(v_i, v_j) = B\log(1 + \frac{P\cdot\ell(v_i,v_j)}{N_0+\sum_{v_k\in A(i)} P\cdot\ell(v_k,v_j)})$, where N_0 is the ambient noise power, $A(i)$ is the set of nodes that transmit when v_i is scheduled. Let d_{ij} be the Euclidean distance between v_i and v_j. Let the power attenuation function be $\ell(v_i, v_j)$. For HEN, let $\ell(v_i, v_j) = \min\{1,\ d_{ij}^{-\alpha}\}$ with $\alpha > 2$ and $N_0 > 0$.

Notations: Throughout this paper, for a 2-dimension line segment $L = uv$, $|L|$ represents the Euclidean distance between u and v; for a discrete set U, $|U|$ represents its cardinality. For a continuous region $\mathcal{A}$, we use $\|\mathcal{A}\|$ to denote its area; for a tree $\mathcal{T}$ (or a forest $\mathcal{F}$) , we use $\|\mathcal{T}\|$ (or $\|\mathcal{F}\|$) to denote its total Euclidean edge length. To simplify the description, let $\theta(n) : [\theta_1(n), \theta_2(n)]$ represent $\theta(n) = \Omega(\theta_1(n))$ and $\theta(n) = O(\theta_2(n))$; and let $\theta(n) : (\theta_1(n), \theta_2(n)]$ represent $\theta(n) = \omega(\theta_1(n))$ and $\theta(n) = O(\theta_2(n))$.

III. Main Results

In this paper, we design three types of strategies for a given *hybrid wireless network*, *i.e.*, *hybrid strategy*, *ordinary ad hoc strategy* and *BS-based strategy*.

1) *Hybrid strategy* use a specific routing and scheduling scheme to let receivers(or source nodes) communicate with central base stations in corresponding subregion, in particular, we can use the other ordinary wireless nodes in same subregion to relay data;
2) *Ordinary ad hoc strategy* will *not* use any base station, in other words, we treat the hybrid network as a pure ad hoc network assuming there are no base stations;
3) *BS-based strategy* can only allow receivers(or source nodes) to communicate with base stations in corresponding subregion *directly*, *e.g.* we do not allow any relay nodes in each subregion.

Please see Fig 1 for illustration.

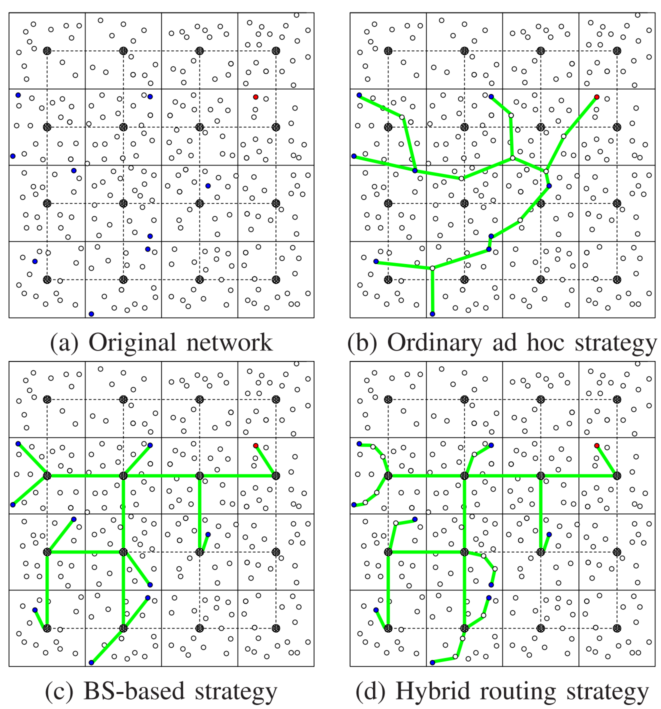


Fig. 1. Illustration of three routing strategies. We use the red node to denote a source node and blues nodes to denote its n_d receivers.

According to the different scenarios in terms of m, n and n_d, we select the optimal scheme from the three categories of strategies, and derive the achievable multicast throughput based on the optimal scheme. The multicast throughput derived by them are presented in Theorem 6, Theorem 7 and Theorem 8, respectively. Combining them, we can obtain the achievable throughput for HEN.

A. Optimal Decision

Theorem 1: Combing three types of strategies, the optimal decision is made as follows.

Case 1: When m: $[1, n/(\log n)^{\alpha+1}]$

1) If $\begin{cases} n_d : [1, n/(\log n)^{3\alpha+2}] \text{ and} \\ m : [\sqrt{nn_d \cdot (\log n)^{\alpha}}, n/(\log n)^{\alpha+1}] \end{cases}$
   The *hybrid strategy* is adopted, by which the throughput is of order $\Omega(\frac{m}{n \cdot n_d}(\log n)^{-\frac{\alpha}{2}})$.
2) Otherwise, the *ordinary ad hoc strategy* is adopted. The throughput is described in Theorem 7.

Case 2: When m: $[n/(\log n)^{\alpha+1}, n/\log n]$

1) If $\begin{cases} n_d : [1, n/(\log n)^{\alpha+2}] \text{ and} \\ m : [\sqrt{nn_d \cdot (\log n)^{\alpha}}, n/\log n] \end{cases}$
   The *hybrid strategy* is adopted, by which the throughput is of order $\Omega(\frac{m}{n \cdot n_d}(\log n)^{-\frac{\alpha}{2}})$.
2) Otherwise, the *ordinary ad hoc strategy* is adopted. The throughput is described in Theorem 7.

Case 3: When m: $[n/\log n, n]$, the *BS-based strategy* is adopted. The throughput is described in Theorem 8.

B. Discussion for results

1) Generality of the results: Due to the generality of multicast sessions, *i.e.*, unicast and broadcast can be regarded as the specific cases of multicast, our result can unify the throughput for unicast and broadcast by letting $n_d = 1$ and $n_d = n - 1$, respectively. However, when we specialize to unicast throughput, *i.e.*, let $n_d = 1$, there is indeed a gap of factor $(\log n)^{-\frac{\alpha}{2}}$ between our result and the result in [15]. In fact, for the routing of [15], the ordinary nodes in each subregion access to the corresponding base station via the *connectivity paths* defined in Section V of this paper. Unlike in *dense networks*, the *connectivity paths* in *extended networks* can only sustain a rate of order $\Omega((\log n)^{-\frac{\alpha}{2}})$ instead of the constant rate as stated in Lemma 5 of [15]. We believe, the mistake in Lemma 5 of [15] lead to the gap between our results for unicast throughput and their results.

2) Analysis of bottleneck: Same as most existing work for the capacity of hybrid networks, we also assume the links between base stations and ordinary wireless nodes (we call such links *B-O links*) have no difference from those between ordinary wireless nodes. While, in the analysis of bottlenecks on three types of strategies (Section V), we find for most cases in terms of m and n_d, the bottlenecks locate on *B-O links*. Therefore, if the bandwidth of *B-O links* can be increased, the throughput for the whole network should possibly be enhanced. Hence, when we consider the *hybrid strategies*, we designedly derive the throughput without taking the possible bottlenecks on the *B-O links* into account. (Please see detail in Theorem 2 and Theorem 4.) We deem that these results could be used when some new assumptions are made for the *B-O links*.

3) Matching upper bounds: As far as we know, even for wireless ad hoc networks, there are still no matching upper bounds and lower bounds for multicast capacity under *Gaussian Channel model*. The same question holds for *hybrid networks* case. Then, it is also an interesting issue to be studied as our further work.

IV. Technical Preparations

A. Probability inequality

Lemma 1 (Chebyshev's Inequality): Let X be a random variable, then $\Pr(|X - \mu| \geq \epsilon) \leq \text{Var}(X)/\epsilon^2$, where $\mu = E(X)$, $\text{Var}(X)$ is the variance of X, and $\epsilon > 0$.

In the following analysis, we often need to prove the uniform convergence in the probability of some events. Vapnik-Chervonenkis Theorem [19] is usually exploited to prove the uniform convergence, as in [5], [11], [13]. When the deployment region $\mathcal{A}$ is partitioned into a lattice consisting of subsquares that act as Voronoi cells, the exponent tails of probability bound can be equally used to prove the uniform convergence of some probability.

Lemma 2 (Tails of Chernoff bound): Let X be a Poisson random variable with parameter λ. Then

$$\Pr(X \geq x) \leq e^{-\lambda}(e\lambda)^x/x^x, \quad \text{for } x > \lambda. \tag{1}$$

$$\Pr(X \leq x) \leq e^{-\lambda}(e\lambda)^x/x^x, \quad \text{for } 0 < x < \lambda. \tag{2}$$

B. Euclidean spanning tree

Partition the square $\mathcal{A}(a)$ into $\rho \leq m$ subsquares called *subregions* while ensuring that there is one base station at the center of each *subregion*, where a is the area of the deployment square region. Notice that one subregion may contain more than one base stations, but we only need to use the central one in our proposed routing scheme. For each multicast session $\mathcal{M}_k$, $k = 1, 2, \cdots n_s$, we denote the *spanning set* as $U_k = \{v_k\} \cup \{v_{k_1}, v_{k_2}, \cdots v_{k_{n_d}}\}$, where v_k is the source node and the nodes in the latter set are the destinations of v_k. Let $U_k^\iota = \{v_{k_1}^\iota, v_{k_2}^\iota, \cdots, v_{k_t}^\iota\}$ denote a subset of U_k to represent the set of nodes contained in the subregion S_ι, where $U_k = \bigcup U_k^\iota$ and $U_k^{\iota_1} \cup U_k^{\iota_2} = \varnothing$ for any $\iota_1 \neq \iota_2$. Let $\tilde{U}_k^\iota = U_k^\iota \cup \{b_\iota\}$, where b_ι denotes the base station that is placed at the center of subregion S_ι. Then, we can build the Euclidean spanning tree (EST) based on every set $\tilde{U}_k^\iota$ using the method in [12], [13]. Denote those ESTs as $\text{EST}(\tilde{U}_k^\iota)$, $1 \leq \iota \leq \varphi_k$, where φ_k is a random variable representing the number of *occupied* subregions, *i.e.*, those containing at least one ordinary node in U_k. Notice that for each $\tilde{U}_k^\iota$ except for that one including v_k (denoted as $\tilde{U}_k^{\iota_o}$), b_ι acts as the root of EST; for $\tilde{U}_k^{\iota_o}$, v_k acts as the root of EST.

It is the complement issue of occupancy question [3] to consider the random variable φ_k, *i.e.*, the number of occupied cells. Suppose that $n_d + 1$ balls are randomly distributed into ρ cells. Assume that each ball has an equal chance of being distributed to each cell. Let $\bar{\varphi}_k$ be the number of cells remaining empty. Hence, $\varphi_k = \rho - \bar{\varphi}_k$. By *occupancy* *theory* [3], the probability distribution of φ_k is given by $\Pr(\varphi_k = z) = \Pr(\bar{\varphi}_k = \rho - z) = C_\rho^z \sum_{i=1}^{z} (-1)^i C_z^i \left(\frac{z-i}{\rho}\right)^{n_d+1}$ where C_ρ^z is the binomial coefficient equal to the number of combinations of z items selected from ρ items. We necessarily pursue the uniform bound of $\varphi_k,\ k = 1, 2, \cdots n_s$.

Define the random variables $\varphi_{max} = \max_k\{\varphi_k\}$ and $\varphi_{min} = \min_k\{\varphi_k\}$. Much research has been implemented to the tail bounds for occupancy ([9]). However, since we concentrate on the lower bounds on multicast capacity, we only need the following straightforward upper bound on φ_{max} (Lemma 3), while noticing that we should use the tail bounds for occupancy to lowerbound φ_{min} when we study the upper bound on multicast capacity.

Lemma 3: $\varphi_{max} = \max_k\{\varphi_k\} = O(\min\{n_d, \rho\})$, *w.h.p.*

Next, we recall an result on the total length of the EST based on a given set of nodes.

Lemma 4: Using the method in [12], [13] to build the EST spanning the set of nodes U, denoted as $\text{EST}(U)$, we have $\|\text{EST}(U)\| \le 2\sqrt{2}\sqrt{|U|} \cdot \sqrt{a}$.

Denote the forest consisting of all $\text{EST}(\tilde{U}_k^\iota)$ $(1 \le \iota \le \varphi_k)$, as $\mathcal{F}_k$. Then, we have

Lemma 5: The total Euclidean edge length of $\mathcal{F}_k$, *i.e.*, $\|\mathcal{F}_k\|$, is *w.h.p.*of order $O(\frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sqrt{n_d \cdot \min\{n_d, \rho\}})$, for any k, $1 \le k \le n_s$.

Proof: Denote the number of vertexes of $\text{EST}(U_k^\iota)$ as x_k^ι and denote the number of vertexes of $\text{EST}(\tilde{U}_k^\iota)$ as $\tilde{x}_k^\iota$ where $1 \le \iota \le \varphi_k$ and $1 \le k \le n_s$. Obviously, $\tilde{x}_k^\iota = x_k^\iota + 1$. According to Lemma 4, $\|\text{EST}(U_k^\iota)\| = O(\sqrt{x_k^\iota - 1} \cdot \frac{\sqrt{a}}{\sqrt{\rho}})$. Hence, there exists a constant κ_1 such that

$$\sum_{\iota=1}^{\varphi_k} \|\text{EST}(U_k^\iota)\| \le \kappa_1 \cdot \frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sum_{\iota=1}^{\varphi_k} \sqrt{x_k^\iota - 1}$$

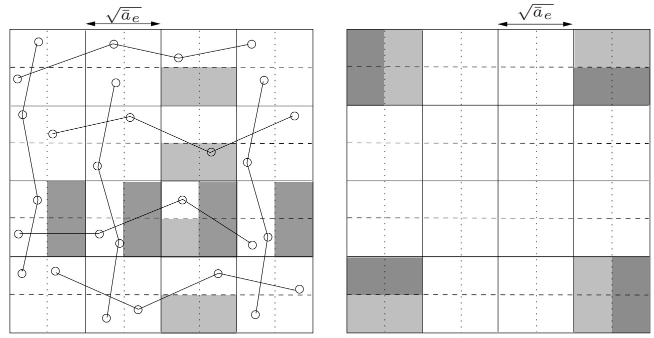
By Cauchy-Schwartz Inequality, we have

$$\sum_{\iota=1}^{\varphi_k} \sqrt{x_k^\iota - 1} \le \sqrt{\varphi_k \sum_{\iota=1}^{\varphi_k} (x_k^\iota - 1)} \le \sqrt{\varphi_k (n_d - \varphi_k)}$$

Since $\|\text{EST}(\tilde{U}_k^\iota)\| \le \|\text{EST}(U_k^\iota)\| + \frac{\sqrt{2a}}{\sqrt{\rho}}$, there is a constant κ_2 such that $\|\mathcal{F}_k\| \le \frac{\sqrt{a}}{\sqrt{\rho}} \cdot (\kappa_1 \sqrt{\varphi_k \cdot (n_d - \varphi_k)} + \kappa_2 \cdot \varphi_k)$. Then, $\|\mathcal{F}_k\| = O(\frac{\sqrt{a}}{\sqrt{\rho}} \cdot \sqrt{n_d \varphi_k})$. Combing with Lemma 3, we complete the proof. ■

C. Result on bond percolation model [4]:

Let $\mathbb{B}(h, p)$ denote a box with side length h embedded in the square lattice. We call a path consisting of only open edges (bonds) *open path*. For a given $\kappa > 0$, we partition the lattice graph $\mathbb{B}(h, p)$ into horizontal (vertical) rectangle slabs with the horizontal (or vertical) width of h and the vertical (horizontal) width of $\kappa \log h - \epsilon(h)$, denoted as R_i^h (or R_i^v). We can choose ε_h as the smallest value such that the number of rectangle slabs $h/(\kappa \log h - \epsilon(h))$ is an integer. It is obvious that $\epsilon(h) = o(1)$ as $h \to \infty$ [4]. Denote the number of edge-disjoint *open paths* in slab R_i^h (or R_i^v) as N_i^h (or N_i^v). Let $N^h = \min_i N_i^h$, $N^v = \min_i N_i^v$. Then,



(a) connectivity paths (b) TDMA scheduling

Fig. 2. (a) The polygonal chains represent the *connectivity paths*. There are at least $\frac{\theta}{2} \log n$ *connectivity paths* in each column (or row). The shaded rectangles represent the *half-cells*. (b) The shaded cells can be scheduled simultaneously in a 9-TDMA scheme. Each time slot can be further divided into four subslots, and the four *half-cells* in each cell are scheduled one out of four subslots.

Lemma 6: ([4]) For any constant $\kappa > 0$ and $p \in (\frac{5}{6}, 1)$ satisfying $2 + \kappa \log(6(1-p)) < 0$, there is a $\delta(\kappa, p)$ such that

$$\lim_{h\to\infty} \Pr(N^h \ge \delta \log h) = 1; \lim_{h\to\infty} \Pr(N^v \ge \delta \log h) = 1.$$

D. Bottleneck principle:

If the adopted strategy is of hierarchical structure, then the bottleneck determines the final throughput derived by the whole phase of multicast strategy. That is,

Lemma 7: The achievable throughput derived by multicast strategy $\Im$ is of $\Lambda = \min\{\Lambda_j;\ j = 1, 2, \cdots, \tau\}$, where we assume that the routing scheme consists of τ phases and assume Λ_j is the throughput in phase j.

V. Multicast Strategy and Throughput

We design three types of multicast strategies, *i.e.*, *hybrid strategy*, *ordinary ad hoc strategy* and *BS-based strategy*, to obtain the achievable multicast throughput for *hybrid extended network* (HEN). A novel technique called *parallel transmission scheduling* is introduced. The assumption is reclaimed that the bottleneck of the whole routing does not locate on the links between base stations (BSs) since they are connected using high bandwidth wired network. However, the links between BSs and ordinary nodes are possible, actually often, becoming the bottleneck of the whole routing. As mentioned before, for the simplicity of analysis, we partition $\mathcal{A}(n)$ into ρ, $\rho \le m$, *subregions* with side length $\frac{\sqrt{n}}{\sqrt{\rho}}$, ensuring there is one base station at the center of each subregion. Notice that there may be more than one base station located at same subregion, but we are only interested at the central one. In the following context, we denote the base station located at the center of subregion S_ι by b_ι.

A. Hybrid Strategy for HEN

The *hybrid strategies* can be further classified into two optional strategies called *connectivity strategy* and *percolation strategy* respectively.

1) Connectivity Strategy: We state that the *connectivity strategy* can be applied when $\rho = O(n/\log n)$. We denote *connectivity strategy* by $\bar{\Im}_e$, and the routing and wireless transmission scheduling by $\bar{\Im}_e^r$ and $\bar{\Im}_e^t$ respectively. Divide $\mathcal{A}(n)$ into subsquares with area $\bar{a}_e = 2\theta \cdot \log n$, where θ is a constant satisfying $\theta > \frac{2}{1-\ln 2}$. We call those subsquares *connectivity cells*. Furthermore, we separate each cell into halves horizontally (or vertically) called horizontal (or vertical) *half-cells*. Then, we have

Lemma 8: With high probability, there are at most $2\theta \cdot \log n$ and at least $\frac{\theta}{2} \cdot \log n$ ordinary nodes in every half-cell.

Proof: According to Lemma 2 and union bounds, Lemma can be proved when $\theta > \frac{2}{1-\ln 2}$. ■

Routing scheme $\bar{\Im}_e^r$: We propose Algorithm 1 to construct the multicast routing tree $\mathcal{T}(U_k)$ for multicast session $\mathcal{M}_k$. For each edge $u_i u_j \in \mathrm{EST}(\tilde{U}_k^\iota)$, $1 \le \iota \le \varphi_k$, we use Manhattan routing to realize it. Notice that each hop in Manhattan routing connects two nodes belonging to two adjacent *connectivity cells* but nonadjacent horizontal (or vertical) *half-cells*, which ensures that the Euclidean length of each hop is at most $\frac{\sqrt{13}}{2}\sqrt{\bar{a}_e}$ and at least $\frac{1}{2}\sqrt{\bar{a}_e}$. We call such paths *connectivity paths*. According to Lemma 8, there are at least $\frac{\theta}{2}\log n$ *connectivity paths* in each slab of size $\sqrt{\bar{a}_e} \times \sqrt{n}$. Hence, we can allocate the total traffic of each slab to such $\frac{\theta}{2}\log n$ *connectivity paths* averagely.

Algorithm 1 Connectivity Routing Scheme $\bar{\Im}_e^r$

Input: $\mathrm{EST}(\tilde{U}_k^\iota)$, $1 \le \iota \le \varphi_k$.
Output: A multicast routing tree $\mathcal{T}(U_k)$.

```
for each EST(Ũ_k^ι) do
    for each link u_i u_j in EST(Ũ_k^ι) do
        Connect u_i and u_j using Manhattan routing:
        Denote the intersection point of the horizontal line
        through u_i and the vertical line through u_j as p_{i,j},
        and denote the nearest node to point p_{i,j} as u_{i,j};
        choose randomly a node in each half-cell passed
        by u_i u_{i,j} and u_j u_{i,j}, and connect alternately those
        nodes, as illustrated in Fig.2(a).
    end for
    Merge the same edges (hops) and remove the circles
    that have no impact on the connectivity of EST(Ũ_k^ι),
    we obtain the multicast tree T(U_k^ι).
end for
Based on the forest consisting of the constructed trees,
i.e., T(U_k^ι) (1 ≤ ι ≤ φ_k), we obtain the final multicast
tree T(U_k) by building an EST spanning the set of base
stations b_ι (1 ≤ ι ≤ φ_k).
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Transmission scheduling $\bar{\Im}_e^t$: We adopt a 9-TDMA scheme, and further divide each time slot into 4 equal *subslots* during which we schedule in turn the four *half-cells* of each cell (Fig.2(b)). The main technique called *parallel transmission scheduling* used here is: In each activated subslot, we schedule simultaneously $\frac{\theta}{2} \cdot \log n$ parallel links (the existence ensured by Lemma 8) instead of scheduling only one link in the previous work. We further prove that

Lemma 9: By the *parallel transmission scheduling* $\bar{\Im}_e^t$, the rate along each *connectivity path* can be sustained of order $\Omega((\log n)^{-\frac{\alpha}{2}})$.

Proof: Considering any link in any time slot, since the length of the link is at least $\frac{1}{2}\sqrt{\bar{a}_e}$, we obtain that the sum of interferences to the receivers is bounded as:

$$\begin{aligned} I(n) &\le P \cdot (\tfrac{\theta}{2}\log n - 1) \cdot \ell(\tfrac{1}{2}\sqrt{\bar{a}_e}) \\ &\quad + \sum_{i=1}^{n} 8i \cdot (\tfrac{\theta}{2}\log n) \cdot P \cdot \ell(\tfrac{3i-2}{2}\sqrt{\bar{a}_e}) \\ &\le P \cdot (\tfrac{2}{\theta})^{\frac{\alpha}{2}-1} \cdot (\log n)^{1-\frac{\alpha}{2}} \cdot \left(1 + \lim_{n\to\infty} \sum_{i=1}^{n} \tfrac{8i}{(3i-2)^\alpha}\right) \end{aligned}$$

The last limitation obviously converges when $\alpha > 2$, thus $I_n = o(1)$. Since the length of every hop is at most $\frac{\sqrt{13\bar{a}_e}}{2}$, we have the signal $S(n)$ at the receiver can be bounded as $S(n) \ge (\frac{13}{2} \cdot \theta)^{-\frac{\alpha}{2}} \cdot P \cdot (\log n)^{-\frac{\alpha}{2}}$. By $\alpha > 2$ and $N_0 > 0$, we have $\frac{S(n)}{N_0 + I(n)} = (\log n)^{-\frac{\alpha}{2}} \to 0$, hence each link can sustain a rate of $\Omega((\log n)^{-\frac{\alpha}{2}})$. ■

Throughput derived by $\bar{\Im}_e$: Firstly, we consider the relay burden of each *connectivity path*.

Lemma 10: By the routing scheme $\bar{\Im}_e^r$, the relay burden of each cell is at most of order

$$\bar{L}_e^r = \begin{cases} O(n_d\sqrt{n}/\sqrt{\rho \log n}) & \text{when } n_d : [1,\ \rho] \\ O(\sqrt{n n_d}/\sqrt{\log n}) & \text{when } n_d : [\rho,\ n/\log n] \\ O(n_d) & \text{when } n_d : [n/\log n,\ n] \end{cases}$$

Proof: Given a node $\bar{v}_t^*$ on a *connectivity path*, define the number of multicast sessions routed through $\bar{v}_t^*$ as a random variable $\bar{\xi}_t$. We finally consider the uniform upper bound $\bar{\xi}$ of $\bar{\xi}_t$ for every node. Define an event $\bar{E}_e^r(k,t)$: The multicast session $\mathcal{M}_k$ passes through $\bar{v}_t^*$. Obviously, if $\bar{E}_e^r(k,t)$ happens then there exists an edge $u_i u_j \in \mathcal{F}_k$ that is routed through $\bar{v}_t^*$, *i.e.*, $u_i u_{i,j}$ or $u_{i,j} u_j$ passes through $\bar{v}_t^*$. Since there exists a constant ϱ_1 such that

$$|u_i u_{i,j}| \le |u_i p_{i,j}| + \varrho_1 \cdot \sqrt{\bar{a}_e}, \quad |u_{i,j} u_j| \le |p_{i,j} u_j| + \varrho_1 \cdot \sqrt{\bar{a}_e}$$

and for $|u_i p_{i,j}| + |p_{i,j} u_j| \le \sqrt{2}|u_i u_j|$, we have

$$\begin{aligned} &\Pr(\bar{E}_e^r(k,t)) \\ &\le \tfrac{1}{\frac{\theta}{2}\cdot\log n} \cdot \tfrac{\sqrt{\bar{a}_e}}{n} \cdot \textstyle\sum_{u_i u_j \in \mathcal{F}_k} (|u_i u_{i,j}| + |u_{i,j} u_j| + 4\sqrt{\bar{a}_e}) \\ &\le \tfrac{2}{\theta \log n} \left(\tfrac{(4+2\varrho_1)(n_d+\varphi_k)\bar{a}_e}{n} + \tfrac{\sqrt{2\bar{a}_e}}{n} \cdot \textstyle\sum_{u_i u_j \in \mathcal{F}_k} |u_i u_j| \right) \\ &\le \tfrac{1}{n} \cdot (\kappa_3 \cdot n_d + \tfrac{4}{\sqrt{\theta \cdot \log n}} \cdot \|\mathcal{F}_k\|) \\ &\le \tfrac{1}{n} \cdot \left(\kappa_3 \cdot n_d + \tfrac{\kappa_4}{\sqrt{\log n}} \cdot \sqrt{\tfrac{n \cdot n_d \cdot \min\{n_d, \rho\}}{\rho}} \right) \end{aligned}$$

where κ_3 and κ_4 are some constants and the last inequality is true according to Lemma 5. Thus, an upper bound of $\bar{\xi}_t$, denoted as $\bar{\eta}_t$, follows Poisson with

$$\bar{\lambda}_e = \frac{n_s}{n} \left(\kappa_3 \cdot n_d + \kappa_4 \sqrt{\frac{n \cdot n_d \cdot \min\{n_d, \rho\}}{\rho \cdot \log n}} \right).$$

Hence, by union bounds, we have

$$\Pr(\bar{\xi} > \sigma\bar{\lambda}_e) \le \frac{1}{\bar{a}_e} \cdot \Pr(\bar{\xi}_t > \sigma\bar{\lambda}_e) \le \frac{n}{2\log n} \Pr(\bar{\eta}_t > \sigma\bar{\lambda}_e)$$

According to Lemma 2, for $\sigma > 1$, $\Pr(\bar{\eta}_t > \sigma\bar{\lambda}_e) \leq (\frac{e^{\sigma-1}}{\sigma^\sigma})^{\bar{\lambda}_e}$. Since $n_s = \Theta(n)$ and $\bar{\lambda}_e = \Omega(\log n)$, we can choose σ satisfying $\frac{e^{\sigma-1}}{\sigma^\sigma} < 1$ (*e.g.* let $\sigma = e$), by which we get

$$\Pr(\bar{\xi} > \sigma\bar{\lambda}_e) = O(1/\log n) \to 0, \text{ as } n \to 0.$$

Then, the relay burden of every node on *connectivity paths* is of order $O(\bar{\lambda}_e)$, which completes the proof. ∎

Combining Lemma 9 and Lemma 10, we can easily obtain Theorem 2.

Theorem 2: When $\rho = O(n/\log n)$, by the strategy $\bar{\Im}_e$ without taking the bottlenecks on BSs into account, the per-session multicast throughput for HEN can achieve order

$$\bar{\Lambda}_e^{\bar{r}_b} = \begin{cases} \Omega((\log n)^{\frac{1-\alpha}{2}} \cdot \frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when } n_d : [1,\ \rho] \\ \Omega((\log n)^{\frac{1-\alpha}{2}} \cdot \frac{1}{\sqrt{nn_d}}) & \text{when } n_d : [\rho,\ n/\log n] \\ \Omega((\log n)^{-\frac{\alpha}{2}} \cdot \frac{1}{n_d}) & \text{when } n_d : [n/\log n,\ n] \end{cases}$$

In the following context we will consider the possible bottleneck that may happen on BSs. Under the strategy $\bar{\Im}_e$, all source nodes in some subregion S_ι will send data to the base station b_ι as long as some receiver node(s) falling outside of S_ι. Thus, the base station may become the bottleneck of the network when the number of source nodes exceeds some value. With the increasing number of source nodes inside one subregion, if most of source nodes have some receivers outside the subregion, the base stations may have huge burden, thus become bottlenecks. Using the similar method in Lemma 10, we have,

Lemma 11: The maximum load on the links between BSs and ordinary nodes is of order

$$\bar{L}_e^{r_b} = \begin{cases} O(n \cdot n_d/\rho) & \text{when } n_d : [1,\ \rho] \\ O(n) & \text{when } n_d : [\rho,\ n] \end{cases}$$

Proof: Define an event $\bar{E}^b(k,t)$: The subregion S_t contains a node belonging to U_k. Then, $\Pr(\bar{E}^b(k,t)) \leq \frac{n_d}{\rho}$, for any $t = 1, 2, \cdots, \rho$. Furthermore, define the load of each subregion as a random variable $\bar{\xi}_t^b$. Then, an upper bound of $\bar{\xi}_t^b$, denoted as $\bar{\eta}_t^b$, follows Poisson with $\bar{\lambda}_e^b = n \cdot \frac{n_d}{\rho}$. Considering the cases $\bar{\lambda}_e^b = O(\log\rho)$ and $\bar{\lambda}_d = \Omega(\log\rho)$ respectively, by using union bounds and Lemma 2, we complete the proof. ∎

According to Lemma 9, the capacity of the links between BSs and ordinary nodes is of order $\Omega((\log n)^{-\frac{\alpha}{2}})$. Thus,

Lemma 12: By $\bar{\Im}_e$, the throughput along the wireless links via BSs is of order

$$\bar{\Lambda}_e^{r_b} = \begin{cases} \Omega(\frac{\rho}{n \cdot n_d} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [1,\ \rho] \\ \Omega(\frac{1}{n} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [\rho,\ n] \end{cases}$$

Combing Theorem 2 and Lemma 12, we conclude that the bottleneck of the whole routing $\bar{\Im}_e^r$ lies on the wireless links via BSs. According to Lemma 7, we obtain the throughput achieved by *connectivity strategy*.

Theorem 3: By the *connectivity strategy* $\bar{\Im}_e$, the per-session multicast throughput for *hybrid extended networks* can be achieved of order:

When $m : [1, n/\log n]$,

$$\bar{\Lambda}_e^r = \begin{cases} \Omega(\frac{m}{n \cdot n_d} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [1,\ m] \\ \Omega(\frac{1}{n} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [m,\ n] \end{cases}$$

When $m : [n/\log n, n]$,

$$\bar{\Lambda}_e^r = \begin{cases} \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{\alpha}{2}-1}) & \text{when } n_d : [1,\ n/\log n] \\ \Omega(\frac{1}{n} \cdot (\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [n/\log n,\ n] \end{cases}$$

2) Percolation Strategy: First of all, we state that the *percolation strategy* can apply for the case when $\rho = O(\frac{n}{(\log n)^2})$. We adopt the *percolation strategy* denoted as $\Im_e$. Obviously, the side length of each subregion is of order $\Omega(\log n)$. We divide the region $\mathcal{A}(n)$ into subsquares with area of a constant a_e by inclined lines. We call those subsquares *percolation cells*. A *percolation cells* is open if it is nonempty (occupied). Obviously, the open probability is $p = 1 - e^{-a_e}$. Using the same procedure in [4], we can map this model into a bond percolation model $\mathbb{B}(h,p)$ where $h = \sqrt{n}/\sqrt{2a_e}$ and $p = 1 - e^{-a_e}$. Moreover, we can partition $\mathcal{A}(n)$ into slabs of size $\sqrt{2a_e}(\kappa\log h - \epsilon_h) \times (\sqrt{n}/\sqrt{m})$, where we can make $\frac{\sqrt{n}}{\sqrt{m}\sqrt{2a_e}(\kappa\log h - \epsilon_h)}$ be an integer by adjusting $\epsilon_h = o(1)$. We call those slabs *highway slabs*. Then, by Lemma 6, we have

Lemma 13: For any $\kappa > 0$ and $a_e > \log 6 + 2/\kappa$, there exists a constant $\delta_1(\kappa, a_e)$ such that there are *w.h.p.*at least $\delta_1 \log n$ horizontal (vertical) highways in all *highway slabs*.

Based on Lemma 13, we can divide horizontally (or vertically) each *highway slab* into slices of size $\kappa_5 \times (\sqrt{n}/\sqrt{\rho})$, where $\kappa_5 = \frac{\delta_1}{2\kappa}$ is a constant. Then, we can define a mapping function from the set of highways to the set of slices. In other words, we can ensure that the traffic initiated from each slice is taken charge by a corresponding *highway* and every highway only bear with the traffic initiated from at most one slice.

Routing scheme $\Im_e^r$: Based on every EST$(\tilde{U}_k^\iota)$, $1 \leq \iota \leq \varphi_k$, we realize each link $u_iu_j \in$ EST$(\tilde{U}_k^\iota)$ by two broad phases, *i.e.*, *highway phase* and *connectivity path phase*. By Lemma 8, we can build at least $\frac{\theta}{2}\log n$ disjoint *connectivity paths* in each slab of size $\sqrt{a_e} \times (\kappa \cdot \log h - \epsilon_h)$. Thus, similar to routing scheme $\bar{\Im}_e^r$, we can allocate averagely the traffic initiated by such slabs to at least $\frac{\theta}{2}\log n$ *connectivity paths*. We propose Algorithm 2 to describe the routing scheme in detail.

Transmission scheduling $\Im_e^t$: We use two independent TDMA schemes to schedule transmissions along *highways* and *connectivity paths*. To be specific, we divide a scheduling period into two sub-periods with same size called *highway scheduling* $\Im_e^{t_1}$ and *connectivity path scheduling* $\Im_e^{t_2}$. The two scheduling phases corresponds to the two phases of routing, *i.e.*, *highways phase* $\Im_e^{r_1}$ and *connectivity path phase* $\Im_e^{r_2}$. The scheme $\Im_e^{t_1}$ can be adopted as same as the scheduling of highways in [4]. Then, we have

Lemma 14: By the transmission scheduling $\Im_e^{t_1}$, the rate along *highways* can be achieved of order $\Omega(1)$.

Since we can only ensure that there exists at least one *connectivity path*, instead of *highway*, passing through every BS b_ι, for $1 \leq \iota \leq \varphi_k$ and $1 \leq k \leq n_s$, then similar to *connectivity strategy*, we have

Lemma 15: By the strategy $\Im_e$, the throughput along the wireless links via BSs is of order $\Lambda_e^{r_b} = \bar{\Lambda}_e^{r_b}$ (in Lemma 12).

The scheme $\Im_e^{t_2}$ can be adopted as same as $\bar{\Im}_e^t$. Then, according to Lemma 9, we can obtain,

Algorithm 2 Percolation Routing Scheme $\Im_e^r$

Input: $\text{EST}(\tilde{U}_k^\iota)$, $1 \le \iota \le \varphi_k$.
Output: A multicast routing tree $\mathcal{T}(U_k)$.

for each $\text{EST}(\tilde{U}_k^\iota)$ **do**
  for each link $u_i u_j$ in $\text{EST}(\tilde{U}_k^\iota)$ **do**
    u_i drains the packets into the specific horizontal *highway* along the specific *connectivity path*.
    Packets are carried along the horizontal *highway*, and are carried along the specific vertical *highway*.
    Packets are delivered to u_j from the vertical highway along the specific *connectivity path*.
  end for
  Merge the same edges (hops) and remove the circles that have no impact on the connectivity of $\text{EST}(\tilde{U}_k^\iota)$, we obtain the multicast tree $\mathcal{T}(U_k^\iota)$.
end for
By using the similar method as Line 7 in Algorithm 1, we obtain the final multicast tree $\mathcal{T}(U_k)$ based on the forests consisting of the trees $\mathcal{T}(U_k^\iota)$ $(1 \le \iota \le \varphi_k)$.

Lemma 16: By $\Im_e^{t_2}$, the rate along each *connectivity paths* can be achieved of order $\Omega(\frac{1}{(\log n)^{\alpha/2}})$.

Throughput derived by $\Im_e$: Firstly, we analyze the load of routing paths in *highway phase* and *connectivity path phase*.

Lemma 17: During *highway phase* $\Im_e^{r_1}$, the maximum relay burden of each node on the *highways* is *w.h.p.*of order

$$L_e^{r_1} = \begin{cases} O(\frac{\sqrt{nn_d}}{\sqrt{\rho}}) & \text{when } n_d : [1,\ \rho] \\ O(\sqrt{nn_d}) & \text{when } n_d : [\rho,\ n/(\log n)^2] \\ O(n_d \log n) & \text{when } n_d : [n/(\log n)^2, n/\log n] \\ O(n) & \text{when } n_d : [n/\log n,\ n] \end{cases}$$

Proof: Given a node v_t^* on the *highways*, define the number of multicast sessions routed through v_t^* in *highway phase* $\Im_e^{r_1}$ as a random variable $\xi_t^{r_1}$, and finally we consider the uniform upper bound ξ^{r_1} of $\xi_t^{r_1}$. Define an Event $E_e^{r_1}(k,t)$: The multicast session $\mathcal{M}_k$ passes through v_t^* in phase $\Im_e^{r_1}$. Obviously, if $E_e^{r_1}(k,t)$ happens then there exists an edge $u_i u_j \in \mathcal{F}_k$ that is routed through v_t^* in phase $\Im_e^{r_1}$, in other words, a vertical (or horizontal) line through v_t^* intersect with the segment $u_i u_{i,j}$ (or $u_{i,j} u_j$). Similar to Lemma 10, we have

$$\begin{aligned} &\Pr(E_e^{r_1}(k,t)) \\ &\le \tfrac{\kappa_5}{n} \cdot \textstyle\sum_{u_i u_j \in \mathcal{F}_k} \left(|u_i p_{i,j}| + |p_{i,j} u_j| + 2\sqrt{2a_e}(\kappa \log h - \epsilon_h)\right) \\ &\le \tfrac{\kappa_6}{n} \cdot (n_d \log n) + \tfrac{\kappa_7}{n} \cdot \|\mathcal{F}_k\| \\ &\le \tfrac{1}{n} \cdot \left(\kappa_6 \cdot n_d \cdot \log n + \kappa_8 \cdot \sqrt{\tfrac{n \cdot n_d \cdot \min\{n_d, \rho\}}{\rho}}\right) \end{aligned}$$

where $\kappa_5 \sim \kappa_8$ are some constants and the last inequality is true according to Lemma 5. Thus, an upper bound of $\bar{\xi}_t$, denoted as η_t, follows Poisson with

$$\lambda_e^{r_1} = \frac{n_s}{n}\left(\kappa_6 \cdot n_d \cdot \log n + \frac{\kappa_8}{\rho} \cdot \sqrt{n \cdot n_d \cdot \min\{n_d, \rho\}}\right)$$

Hence, by the similar procedure of Lemma 10, we obtain that the relay burden of every node on the *highways* in phase $\Im_e^{r_1}$ is of order $O(\lambda_e^{r_1})$, which completes the proof. ■

Lemma 18: During *connectivity path phase* $\Im_e^{r_2}$, the maximum relay burden of each node on the *connectivity path* is *w.h.p.*of order $L_e^{r_2} = O(n_d(\log n)^{1/2})$.

Proof: For a given node v_t^{**} on the *connectivity paths*, define the number of multicast sessions routed through v_t^{**} in *connectivity path phase* $\Im_e^{r_2}$ as a random variable $\xi_t^{r_2}$, and finally we consider the uniform upper bound ξ^{r_2} of $\xi_t^{r_2}$. Define an Event $E_e^{r_2}(k,t)$: The multicast session $\mathcal{M}_k$ passes through v_t^{**} in phase $\Im_e^{r_2}$. We can see that if $E_e^{r_2}(k,t)$ happens then there is a node belongs to U_k and locates in a slab of size $\frac{2\sqrt{a_e}}{\theta \cdot \log n} \times (\kappa \cdot \log h - \epsilon_h)$. Hereafter, using a similar procedure in Lemma 17, we can complete the proof. ■

Combining Lemma 14 and 17, we obtain Lemma 19.

Lemma 19: During phase $\Im_e^{r_1}$, the multicast throughput can be achieved of order

$$\Lambda_e^{r_1} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when } n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when } n_d : [\rho,\ n/(\log n)^2] \\ \Omega(\frac{1}{n_d \log n}) & \text{when } n_d : [n/(\log n)^2, n/\log n] \\ \Omega(1/n) & \text{when } n_d : [n/\log n,\ n] \end{cases}$$

Furthermore, combining Lemma 16 and Lemma 18, we can obtain the following lemma.

Lemma 20: During phase $\Im_e^{r_2}$, the multicast throughput can be achieved of order $\Lambda_e^{r_2} = \Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{\alpha+1}{2}})$.

Based on Lemma 19 and Lemma 20, and according to Lemma 7, we obtain Theorem 4.

Theorem 4: When $\rho = O(n/(\log n)^2)$, by the *percolation strategy* $\Im_e$ without taking bottlenecks on BSs into account, the per-session multicast throughput for *hybrid extended networks* can be achieved of order:
When $\rho : [1, (n/(\log n)^{\alpha+1})]$,

$$\Lambda_e^{\bar{r}_b} = \begin{cases} \Omega(\frac{\sqrt{\rho}}{n_d\sqrt{n}}) & \text{when } n_d : [1,\ \rho] \\ \Omega(\frac{1}{\sqrt{nn_d}}) & \text{when } n_d : [\rho,\ \frac{n}{(\log n)^{\alpha+1}}] \\ \Omega(\frac{1}{n_d \cdot (\log n)^{\frac{\alpha+1}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^{\alpha+1}}, n] \end{cases}$$

When $\rho : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}]$, $\Lambda_e^{\bar{r}_b} = \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}})$.

Combing Theorem 4 and Lemma 15, we have

Theorem 5: By the *percolation strategy* $\Im_e$, the per-session multicast throughput for HEN is achieved of order:
When $m : [1, n/\log n]$,

$$\Lambda_e^r = \begin{cases} \Omega(\frac{m}{n \cdot n_d}(\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [1,\ m] \\ \Omega(\frac{1}{n}(\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [m,\ \frac{n}{\sqrt{\log n}}] \\ \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}}) & \text{when } n_d : [\frac{n}{\sqrt{\log n}},\ n] \end{cases}$$

When $m : [n/\log n, n]$,

$$\Lambda_e^r = \begin{cases} \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha}{2}-1}) & \text{when } n_d : [1,\ n/\log n] \\ \Omega(\frac{1}{n}(\log n)^{-\frac{\alpha}{2}}) & \text{when } n_d : [n/\log n,\ \frac{n}{\sqrt{\log n}}] \\ \Omega(\frac{1}{n_d}(\log n)^{-\frac{\alpha+1}{2}}) & \text{when } n_d : [\frac{n}{\sqrt{\log n}},\ n] \end{cases}$$

Furthermore, combing Theorem 3 and Theorem 5, we can get the throughput derived by *hybrid routing strategies*.

Theorem 6: By the *hybrid strategies*, the multicast throughput for HEN is achieved of order Λ_e^r (defined in Theorem 5).

B. Ordinary Ad hoc Strategy for HEN

Different from the previous routing strategy, in *ordinary ad hoc strategy*, we will not use any base station but only the ordinary nodes. In particular, we treat the network as a ordinary Ad hoc network and we construct global multicast trees composed by only ordinary nodes. Similar to *hybrid strategies*, the *ordinary ad hoc strategies* consist of *connectivity strategy* and *percolation strategy*. Indeed, the *ordinary ad hoc strategies* can be regarded as the specific cases of hybrid strategies by removing the technics about BSs. Then, by using a similar procedure in analysis of *hybrid strategies*, we obtain,

Theorem 7: By the *ordinary ad hoc strategies*, the multicast throughput for HEN is achieved of order

$$\begin{cases} \Omega(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Omega(\frac{1}{n_d (\log n)^{\frac{\alpha+1}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}] \\ \Omega(\frac{1}{\sqrt{n n_d} \cdot (\log n)^{\frac{\alpha-1}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Omega(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases}$$

C. BS-based Strategy for HEN

In this routing strategy, we adopt a classical base station (BS) based data transmission, in which sources deliver data to BSs directly during the uplink phase and BSs deliver received data to destinations directly during the downlink phase. Since in any time slot, all wireless links associate with the BSs, then the *parallel transmission scheduling* is disabled. We denote the BS-based strategy as $\tilde{\Im}_e$ and denote the corresponding routing scheme and transmission scheduling as $\tilde{\Im}_e^r$ and $\tilde{\Im}_e^t$, respectively. Different from the previous partition method, here we simply partition $\mathcal{A}(n)$ into m *subregions* with side length $\frac{\sqrt{n}}{\sqrt{m}}$ and each base station is placed at the center of each subregion.

Routing scheme $\tilde{\Im}_e^r$: The routing consists of three phases: *uplink phase* $\tilde{\Im}_e^{r_1}$, *BS-to-BS phase* $\tilde{\Im}_e^{r_2}$ and *downlink phase* $\tilde{\Im}_e^{r_3}$. That is,

1) During the uplink phase, source nodes in subregion S_ι, $\iota = 1, 2, \cdots, m$, transmit the packets to BS b_ι.
2) The BS that receives the packet from source v_k, $k = 1, 2, \cdots, n_s$, delivers it to the BSs placed in the subregions containing the nodes in the set of destination of v_k using BS-to-Bs links.
3) During *downlink phase*, each BS b_ι, $\iota = 1, 2, \cdots, m$, broadcasts the packets to the nodes in subregion S_ι.

Transmission scheduling $\tilde{\Im}_e^t$: It includes three independent phases, *i.e.*, $\tilde{\Im}_e^{t_1}$, $\tilde{\Im}_e^{t_2}$ and $\tilde{\Im}_e^{t_3}$, corresponding to three routing phases. Since the *BS-to-BS phase* is surely not the bottleneck anyway, we only focus on the other two phases. That is,

1) During *uplink phase* $\tilde{\Im}_e^{t_1}$, all BSs b_ι, $\iota = 1, 2, \cdots, m$, receive simultaneously packets from the nodes in S_ι.
2) During *downlink phase* $\tilde{\Im}_e^{t_3}$, all BSs b_ι, $\iota = 1, 2, \cdots, m$, deliver simultaneously packets to the nodes in S_ι.

Lemma 21: By the scheduling $\tilde{\Im}_e^t$, each *subregion* can sustain traffic with a rate of order $\Omega((n/m)^{-\frac{\alpha}{2}})$ during both downlink and uplink.

Proof: Due to the regular location of BSs, for any receiver in a *subregion*, the nearest transmitters outside the *subregion* is faraway with distance of at least $\frac{\sqrt{n}}{2\sqrt{m}}$. Similar to Lemma 9, the sum of interferences to the receivers is bounded as: $I(n) \leq \sum_{i=1}^{m} 8iP\ell(\frac{2i-1}{2} \cdot \frac{\sqrt{n}}{\sqrt{m}}) \leq (\frac{m}{n})^{\frac{\alpha}{2}} \cdot 2^{\alpha} P \sum_{i=1}^{\infty} \frac{8i}{(2i-1)^{\alpha}}$. Thus, $I(n) = O((n/m)^{-\frac{\alpha}{2}})$. While, the signal $S(n)$ can be bounded as $S(n) \geq P \cdot (\frac{\sqrt{n}}{\sqrt{2m}})^{-\alpha} \geq (\frac{\sqrt{10}}{2})^{-\alpha} \cdot P \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}$, *i.e.*, $S(n) = \Omega((n/m)^{-\frac{\alpha}{2}})$. For the assumption $m = O(n)$, we have $I(n) = O(1)$ and $S(n) = O(1)$. Hence, we have $\frac{S(n)}{N_0+I(n)} = O(1)$ and $\log(1 + \frac{S(n)}{N_0+I(n)}) = \Omega((n/m)^{-\frac{\alpha}{2}})$, which completes the proof. ■

Next, we consider the load of BS during the downlink phase or uplink phase. Similar to Lemma 11, we have,

Lemma 22: By the strategy $\tilde{\Im}_e^r$, the load of each base station is of order $\tilde{L}_e^r = \bar{L}_e^{r_b}$.

According to Lemma 21 and Lemma 22, we have

Theorem 8: By the BS-based strategy, the per-session multicast throughput for HEN can be achieved of order

$$\begin{cases} \Omega(\frac{1}{\log m} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when } n_d : [1, \ m \log m/n] \\ \Omega(\frac{m}{n \cdot n_d} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when } n_d : [m \log m/n, \ m] \\ \Omega(\frac{1}{n} \cdot (\frac{n}{m})^{-\frac{\alpha}{2}}) & \text{when } n_d : [m, \ n] \end{cases}$$

D. Integration of Three Types of Strategies

To achieve the optimal throughput, we will select the best strategy according to the different scenarios in terms of m and n_d. Combing Theorem 6, Theorem 7 and Theorem 8, we can obtain the main result in Theorem 1.

VI. Literature Reviews

A. Wireless ad hoc networks

Under threshold-based channel model: Gupta and Kumar [5] studied the *unicast* capacity in *dense networks*, they showed that a scheme of nearest neighbor communication can achieve a throughput of $\Theta(1/\sqrt{n \log n})$. Keshavarz-Haddad et al. [6] studied the broadcast capacity of an arbitrary network, and showed that the per-session broadcast capacity is only of $\Theta(1/n)$. Li et al. [12] showed that, assuming that $n_s = \Omega(\log n_d \sqrt{n \log n / n_d})$ [13], for *random networks*, the per-session capacity of n_s multicast sessions is $\Theta(1/\sqrt{n_d n \log n})$ when $n_d = O(n/\log n)$, and is $\Theta(1/n)$ when $n_d = \Omega(n/\log n)$. Shakkottai et al. [17] designed a routing scheme, called *comb scheme*, and their result can be regarded as a special case of Li's [13].

Under Gaussian Channel model: Franceschetti et al. [4] showed the throughput for *random networks* can be achieved of $\Omega(1/\sqrt{n})$. Zheng [20], [21] proved that the broadcast capacity for *extended networks* is $\frac{1}{n}(\log n)^{-\frac{\alpha}{2}}$. Li et al. [11] showed that, when $n_d = O(\frac{n}{(\log n)^{2\alpha+6}})$ and $n_s = \Omega(n^{\frac{1}{2}+\theta})$, the multicast throughput for random networks can be achieved of $\Omega(\frac{\sqrt{n}}{n_s \sqrt{n_d}})$, where $\theta > 0$ is a constant. Keshavarz-Haddad et al. [7] proposed a technique called *arena* to study upper bounds of capacity. They [8] sketched a scheme and estimate the achievable throughput for *random dense networks*.

B. Hybrid wireless networks

Under threshold-based channel model: Earlier, Liu et al. [14] introduced the model based on the *dense network* in which the base stations are regularly placed and the ad hoc nodes are randomly distributed. While, the case that both base stations and ad hoc nodes are randomly placed in the *dense network* is studied by Kozat and Tassiulas in [10]. Agarwal et al. [1] considered the unicast capacity for hybrid networks under PhIM. Recently, Mao et al. [16] studied the *multicast capacity* for hybrid networks for the case of $m = O(n/\log n)$ that is derived under *threshold-based channel* model, too.

Under Gaussian Channel model: Liu et al. [15] studied the unicast capacity of the wireless ad hoc network with infrastructure. They showed that in a two-dimensional square hybrid wireless network with n ordinary nodes and m base stations, it is necessary that $m = \Omega(\sqrt{n})$ in order to obtain a linear gain of capacity.

VII. Conclusion

We study the achievable multicast throughput for wireless *hybrid extended networks* under *Gaussian Channel model*. Three types of multicast strategies are proposed. Based on our derived achievable multicast throughput for each scheme, we give a optimal decision on selecting one of the three categories of strategies according to the different scenarios of m, n and n_d. To the best of our knowledge, this is the first work that address the multicast routing and scheduling strategy in hybrid wireless networks under Gaussian channel model. A number of interesting questions remain open: How to derive tight upper bound on the network capacity? What kind of strategy need to be implemented if the access link between ordinary wireless nodes and base station is different from the wireless link, *e.g.* it may have larger bandwidth, or if the link between base stations is not wired and its bandwidth is not arbitrary large.

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