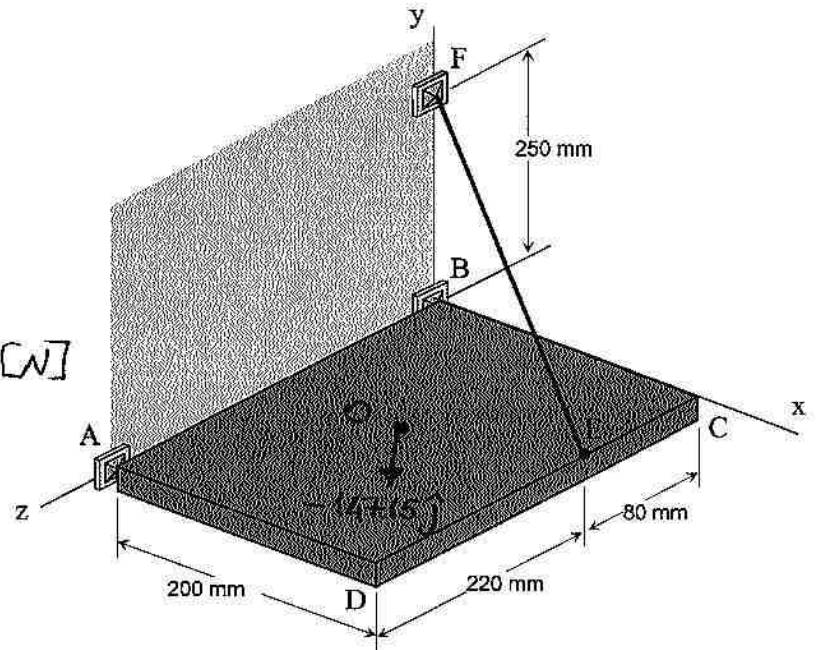


**MMAE 201 MECHANICS OF SOLIDS I****FINAL EXAM (Solutions)**

Q1 – The rectangular plate shown has a mass of 15 kg and is held in the position shown by the ball-and-socket joints A and B and cable EF. Determine the tension in the cable in terms of lb. (Hint: don't forget the weight of the plate) [25 points].



$$\bullet \quad \underline{W} = -15 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \hat{j} = \underline{-147.15 \hat{j}} \text{ [N]}$$

$$\bullet \quad \underline{T}_{EF} = \underline{\Delta}_{EF} \cdot T_{EF}$$

$$= \frac{-0.20 \hat{i} + 0.25 \hat{j} - 0.08 \hat{k}}{\sqrt{0.2^2 + 0.25^2 + 0.08^2}} \cdot T_{EF}$$

$$= T_{EF} (-0.606 \hat{i} + 0.757 \hat{j} - 0.242 \hat{k})$$

If the system is in equilibrium the moment about BA-axis should be zero:

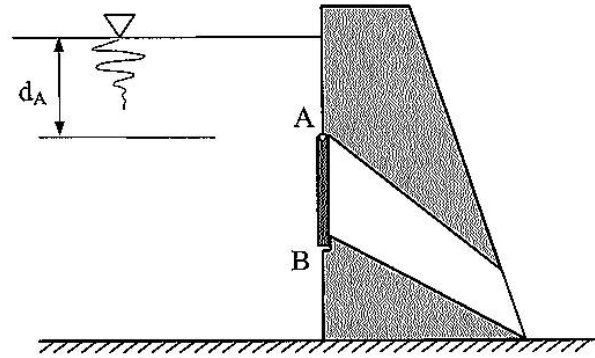
$$\sum M_{BA} = \underline{\Delta}_{BA} \cdot (\underline{r}_{BE} \times \underline{T}_{EF} + \underline{r}_{BO} \times \underline{W}) \quad \leftarrow \begin{cases} \underline{r}_{BE} = 0.20 \hat{i} + 0.08 \hat{k} \text{ [m]} \\ \underline{r}_{BO} = 0.10 \hat{i} + 0.15 \hat{k} \text{ [m]} \\ \underline{\Delta}_{BA} = \frac{\underline{BA}}{|\underline{BA}|} = \frac{0.3 \hat{k}}{0.3} = \hat{k} \end{cases}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 0.2 & 0 & 0.08 \\ -0.606 & 0.757 & -0.242 \end{vmatrix} \cdot T_{EF} + \begin{vmatrix} 0 & 0 & 1 \\ 0.1 & 0 & 0.15 \\ 0 & -147.15 & 0 \end{vmatrix} = 0 \text{ [N}\cdot\text{m]}$$

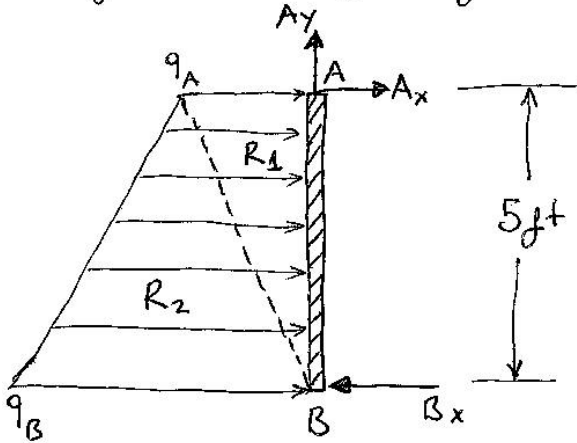
$$0.1514 T_{EF} + (-14.715) = 0 \quad \Rightarrow \quad T_{EF} = \underline{97.2 \text{ N}} = \frac{97.2 \text{ N}}{4.448 \text{ N/lb}} = 21.85 \text{ lb}$$

$$\boxed{T_{EF} = 21.85 \text{ lb}} *$$

Q2 - The cross-section of a concrete dam is as shown. There is a  $5 \times 5 \text{ ft}^2$  rectangular gate AB built to control water flow into a channel (channel is empty). The gate is held in position by hinges along its top edge A, and rests against a frictionless surface at point B. The depth of point A from the water surface is  $d_A = 15 \text{ ft}$ . Determine the magnitude and direction of reaction at B. Use  $\gamma = 62.4 \text{ lb/ft}^3$  for the specific weight of water. (Hint: consider the free-body diagram of gate AB) [25 points].



free-body diagram of AB:



$$q_A = \frac{P_A}{W} \quad \text{width of the gate (5 ft)}$$
$$q_A = \gamma \cdot d_A \cdot W$$
$$= (62.4 \frac{\text{lb}}{\text{ft}^3}) \cdot (15 \text{ ft}) \cdot (5 \text{ ft})$$
$$\Rightarrow q_A = 4680 \text{ lb/ft}$$

$$q_B = \gamma \cdot d_B \cdot W = (62.4) \cdot (20) \cdot (5)$$
$$\Rightarrow q_B = 6240 \text{ lb/ft}$$

$$R_1 = \frac{1}{2} q_A \cdot (5 \text{ ft}) = \underline{\underline{11,700 \text{ lb}}}$$

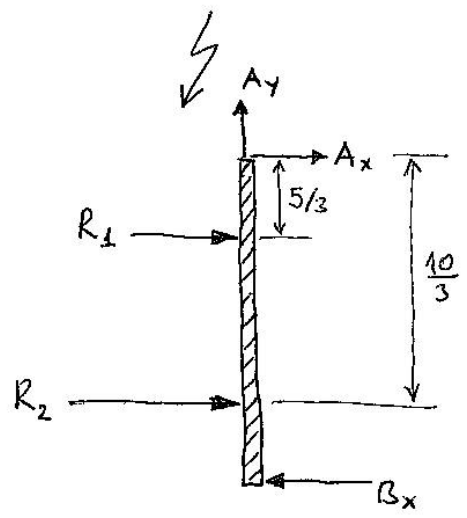
$$R_2 = \frac{1}{2} q_B \cdot (5 \text{ ft}) = \underline{\underline{15,600 \text{ lb}}}$$

Equilibrium equation:

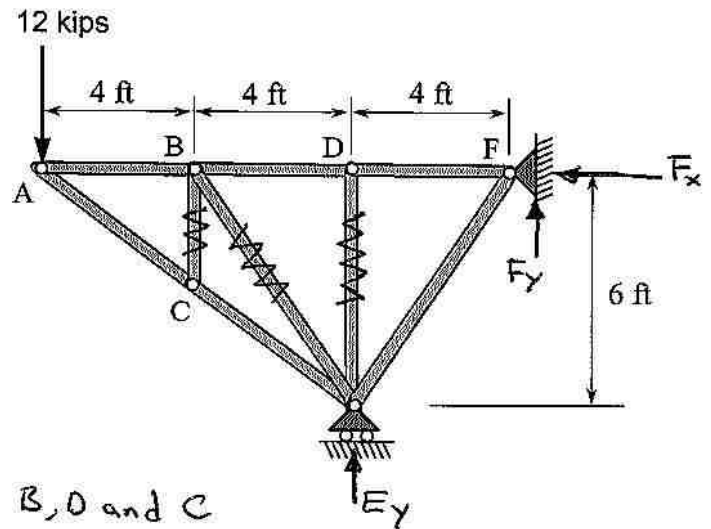
$$\sum M_A = R_1 \left( \frac{5}{3} \right) + R_2 \left( \frac{10}{3} \right) - B_x (5) = 0$$

19,500      52,000

$$\Rightarrow B_x = 14,300 \text{ lb} \quad *$$



Q3 – Determine the force in each member of the truss shown. State whether each member is in tension or compression. (Hint: look for zero-force members) [25 points].



- Note that the members BC, BE and DE are zero-force members

Moreover,  $F_{AB} = F_{BD} = F_{DF}$

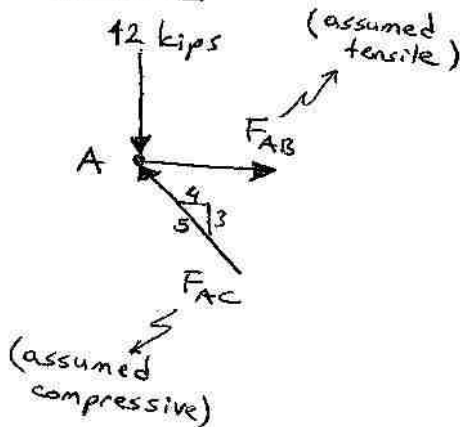
$F_{AC} = F_{CE}$

in order to satisfy equilibrium at pins B, D and C

- Overall equilibrium of truss structure requires :

$\sum M_E = 12 \text{ kips} \times 12 \text{ ft} - E_y \times 4 \text{ ft} = 0 \Rightarrow E_y = 36 \text{ kips}$

Joint A :



$\sum F_x = -\frac{4}{5} F_{AC} + F_{AB} = 0$

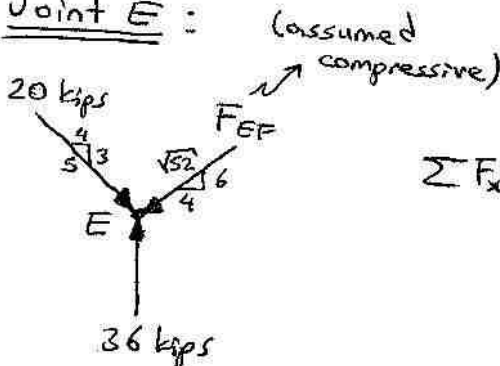
$\sum F_y = -12 + \frac{3}{5} F_{AC} = 0 \Rightarrow F_{AC} = 20 \text{ kips} \quad *C$

$F_{AB} = \frac{4}{5} \cdot 20 \text{ kips} = 16 \text{ kips}$

$F_{AC} = F_{CE} = 20 \text{ kips}$

$F_{AB} = F_{BD} = F_{DF} = 16 \text{ kips} \quad *T$

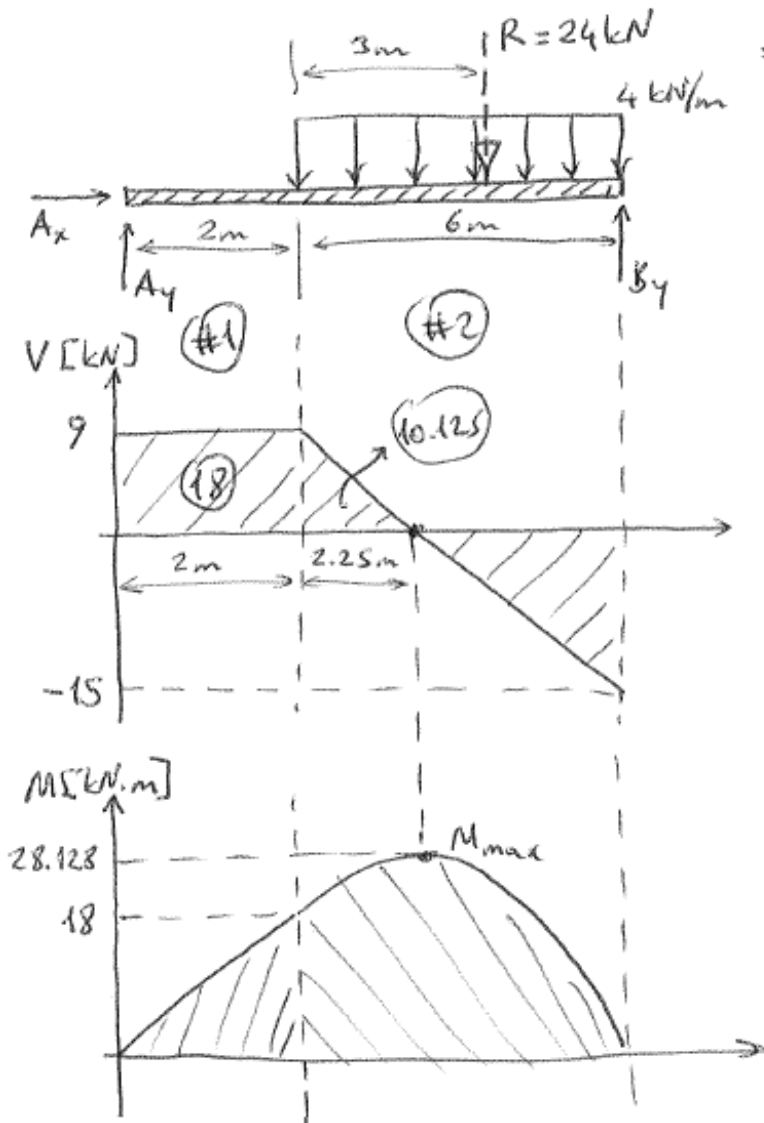
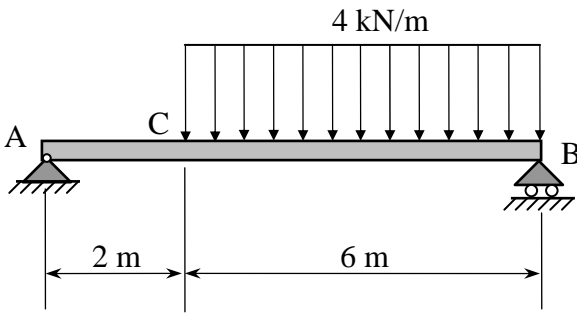
Joint E :



$\sum F_x = \frac{4}{5} \cdot 20 \text{ kips} - \frac{4}{\sqrt{52}} \cdot F_{EF} = 0$

$\Rightarrow F_{EF} = 28.84 \text{ kips} \quad C$

Q4 – For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment. [25 points].



\* From the FBD of entire beam:

$$\sum M_A = 0 = B_y \cdot 8 - 24 \cdot 5$$

$$\Rightarrow B_y = 15 \text{ kN}$$

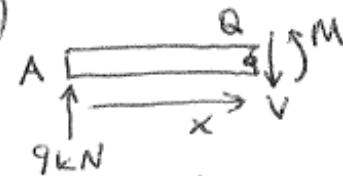
$$\sum F_y = 0 = A_y + B_y - 24$$

$$\Rightarrow A_y = 9 \text{ kN}$$

$$\sum F_x \rightarrow A_x = 0$$

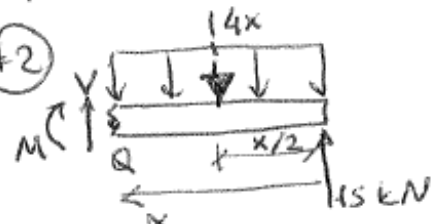
\* Method of sections:

(#1)



$$\sum F_y \rightarrow V = 9 \text{ kN} \quad \sum M_Q \rightarrow M = 9x \text{ [kN}\cdot\text{m]}$$

(#2)



$$\sum F_y \rightarrow V = 4x - 15 \text{ [kN]}$$

$$\sum M_Q \rightarrow M = 15x - 2x^2 \text{ [kN}\cdot\text{m]}$$