

# Static Channel Assignment for Multi-Radio Multi-Channel Multi-Hop Wireless Networks

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**Abstract**—Multi-radio multi-channel multi-hop wireless networks recently have drawn considerable research attention. A number of results have been presented on joint optimization of routing, link scheduling, and channel assignment in such networks. Although dynamic channel switching has some benefits, it also poses a number of challenges in managing the network protocols and causes delay in switching channels. In this paper, we study the *static* channel assignment problem by statically mapping a channel to each radio of the nodes. We first show that it is NP-complete to find a channel assignment such that the resulting network is connected while the number of assigned channels to each node is at most the number of its Network Interface Cards (NICs). Secondly, we give several novel methods that only use a small number of channels while resulting in a connected network. Thirdly, we present a novel method to assign channels such that the network will support a large number of simultaneous transmissions (thus possibly increasing the network throughput). Fourthly, We theoretically analyze the expected number of channels that a node needs to be able to operate so that it can result in a connected network with high probability. Finally, we conduct extensive simulations to study the performances of our algorithms. Our simulations show that our backbone based channel assignment method will give us a connected network without violating the NIC constraints with high probability.

## I. INTRODUCTION

Wireless mesh network is a network implemented over a wireless LAN, and its infrastructure type is decentralized, relatively inexpensive, reliable and resilient since the wireless routers are often static and not powered by batteries. In addition, the access points (routers) seldom change their positions in wireless mesh networks and some wireless nodes have gateway functions to provide the connectivity to the Internet. Therefore, such networks behave almost like wired networks with infrequent topology changes and limited node failures. Due to these good properties and potential applications such as providing commercial Internet access to residents and local businesses, wireless mesh networks have drawn a lot of attention in recent years. For example, wireless mesh networks are being used as the last mile for extending the Internet connectivity for mobile nodes. In addition, some commercial deployments of multi-hop wireless meshes are already in the works. For instance, a number of mesh networks have been

deployed in some cities of the US, including Medford, Oregon, and cities in Minnesota [1].

Unlike traditional wired network, close-by mesh routers share certain wireless channels and the capacity of wireless mesh network will be increased tremendously when extending from a single channel model to a multi-radio, multi-channel, multi-hop model. For example, if two nodes  $v_i$  and  $v_j$  could communicate with each other by channel  $f_1$ , and both nodes have at least one more available network interface card which could operate on another channel  $f_2$ , if  $f_2$  is also available for both nodes,  $v_i$  and  $v_j$  could use both  $f_1$  and  $f_2$  to communicate simultaneously. When such cases are applied to more wireless nodes, the throughput of a wireless network will be increased tremendously.

On the other hand, with the recent fast growing spectrum-based services and devices, the remaining spectrum available for future wireless services is being exhausted, known as the *spectrum scarcity problem*. This is a serious problem in several aspects: the cell phone companies have to spend billions of dollars to bid for a 20-30 MHz 3G spectrum; it is difficult for new innovative technologies to enter the market competitively; it is difficult to provide a certain quality of service for systems operating in unlicensed spectrums. The current fixed spectrum allocation scheme leads to significant spectrum *white spaces* (spectrum that is not used at all for some period of time), where many allocated spectrum blocks are used only in certain geographical areas and/or in brief periods of time. As an example, the Shared Spectrum Company performed spectrum occupancy measurements, and found that the spectrum usages vary dramatically in time, geographic locations, and frequency. For example, there was at least 87% of *white space*, measured in New York City [13]. Thus, a huge amount of precious spectrum (below 5GHz), perfect for wireless communications that is worth billions of dollars, sits there silently.

It is widely agreed that opportunistic or dynamic spectrum usage will significantly mitigate the spectrum scarcity. Recently more and more researchers have been concentrating on how to reuse the licensed spectrums for secondary users when the licensed spectrum is idle or secondary users use licensed spectrums within some special interference temperature [9] such that secondary users interfere with the primary users in the range of interference temperature. However, a series of technologies (topology of a wireless network, routing, link scheduling and so on) become more complex and difficult due to the properties of wireless networks (nodes), such as

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lower power, interferences, mobility and some other physical barriers. In addition, using multiple channels and radios could mitigate such interference and improve the throughput to some extent, but it could not eliminate interference completely. All in all, the performance(capacity) improvement brought by multi-channel and multi-radio will be less effective or even degrade if we do not handle them carefully.

In this paper, we address the problem of static channel assignment in wireless networks with multi-radio and multi-channels under some network models and interference models. Assume that each wireless node  $u$  has a given  $\kappa(u)$  number of network interface cards and a subset of channels this node may operate on. We need to choose a channel for each NIC of every node so that the resulting network has certain properties like being connected, *i.e.*, assigning  $\kappa(u)$  channels to node  $u$ . Here two nodes  $u$  and  $v$  can form a communication link  $(u, v)$  if they share a common assigned channel. We prove that it is NP-hard to decide whether we can assign channels to nodes such that the network is connected while the number of total assigned channels to every node does not exceed the number of NICs it has. We then propose several novel methods for assigning channels. One of our methods is based on nodes clustering: we first partition nodes into a number of clusters and connect these clusters via a backbone; we then find a channel assignment for nodes on the backbone such that the backbone is connected; finally, for each cluster, we perform a local channel assignment such that all nodes in a cluster will form a connected subnetwork. One of the advantages using the cluster based channel assignment is that the method can adapt to the dynamic channel availability. When a node's channel availability changes, we only need to update the channel assignment for one cluster if the node is not on the backbone. If it is on the backbone, we only need to additionally update the channel assignment for backbone nodes, whose amount is far less than the total number of wireless nodes in the whole network. Our extensive simulations show that our backbone based channel assignment method induces a connected network without violating the NIC constraints for all nodes with high probability.

We also study how to assign channels to get a connected network using as few channels as possible. By using fewer channels, we leave more freedom for future channel assignment when new nodes join the network. We also study how to assign channels such that the resulting network supports a large number of simultaneous transmissions, thus increase the network throughput of the resulting network intuitively.

As it is observed in the literature, many network properties enjoy a transition phenomenon. We also show that the network connectivity also has a transition phenomenon with respect to the number of channels available to a node when all other parameters are fixed. Assume that the total number of channels in the network is fixed and each node can only operate on  $s$  channels that are *randomly* chosen. Then we show that the network is connected with high probability if  $s$  is larger than a certain threshold value, depending on the number  $n$  of network nodes, the transmission radius  $r$  of every node, the number  $R$  of available radios of every node, and the deployment region (typically assumed to be a unit-area square or a disk)

The rest of the paper is organized as follows: In section II, we state our network models, interference models and the problems to be studied. We give an overview of related work about multi-radio, multi-channel, multi-hop wireless networks in Section III. In Section IV, we study the transition phenomena which plays a key role for the connectivity of wireless networks. We present our detailed algorithms in Section V, and Section VI describes our simulation studies of proposed algorithms. Finally, Section VII concludes the paper.

## II. PROBLEM FORMULATION

In this paper, we will study *static* channel assignment for a multi-radio, multi-channel, multi-hop wireless network. Our objective is either to minimize the used channels while preserving a connected network, or to maximize a certain network throughput using given radio and channel availability of all nodes.

### A. Network System Models

We first present the network model used throughout this paper. Consider a set  $V = \{v_1, v_2, \dots, v_n\}$  of  $n$  wireless nodes distributed in a plane. We first assume that each node  $v_i$  has a transmission range  $R_T(i)$  that allows only those nodes within distance  $R_T(i)$  from  $v_i$  to receive the signal from  $v_i$  correctly. Here we emphasize that  $\|v_j - v_i\| \leq R_T(i)$  is a necessary but not sufficient condition for two nodes  $v_i$  and  $v_j$  to compose a physical communication link, *e.g.*, such link may not exist due to physical barriers. We assume that each terminal  $v_i$  also has an interference range  $R_I(i)$  such that every unintended receiver would be interfered by the signal from  $v_i$  when it is using the same channel as  $v_i$  does simultaneously. Typically,  $R_T(i) \leq R_I(i) \leq \beta \cdot R_T(i)$ , where  $\beta$  is a constant factor and  $2 \leq \beta \leq 4$  in practice. We use  $G_0 = (V, E)$  to denote the complete communication graph, where  $E$  is the set of all communication links.

In addition, every wireless terminal (node)  $v_i$  has one or more Network Interface Cards and each of them could operate on one or more spectrum radios. Here we use  $I(v_i)$  to denote the number of NICs of a node  $v_i$ . Let  $F = \{f_1, f_2, \dots, f_k\}$  denote the set of  $K$  orthogonal (could be non-orthogonal in certain situations) channels which could be used by all nodes  $V$ . We generally assume that each NIC could only operate on a subset of  $F$  channels due to the hardware constraints, and let  $F(v, i)$  be the set of channels that can be operated by  $i^{th}$  NIC for node  $v$ , where  $1 \leq i \leq I(v)$ . Let  $F(v)$  be the set of channels that can be used by node  $v$  via any of its NICs, *i.e.*,  $F(v) = \bigcup_{1 \leq i \leq I(v)} F(v, i)$ . For each link  $e = (u, v) \in E$ ,  $F(e)$  denotes the set of channels which could be used by  $e$ , that is  $F(e) = F(u) \cap F(v)$ . When  $F(e)$  is empty, it means that the two end-nodes of  $e$  cannot communicate directly over  $e$  under the radio and channel constraints. For simplicity, we abuse the notation a little bit, we use  $G = (V, E)$  to denote the communication graph where  $E$  now denotes all the links  $e = (u, v)$  such that  $u$  and  $v$  can communicate directly with each other under the radio and channel constraints of these two nodes.

## B. Problem Formulation

### Transition Phenomena of Network Connectivity on Channel Availability:

The first question we want to investigate is the effect of channel availability on the connectivity of network  $G$ . Assume that due to designing or environmental constraints, a wireless NIC can only operate on a randomly selected  $c$  channels out of total maximum  $|F| = k$  channels. It is not difficult to see that when  $c$  is sufficiently smaller than  $k$ , the network is more likely to be disconnected, and when  $c$  is sufficiently close to  $k$ , the network is more likely to be connected (when the graph  $G_0$  is connected). We thus would like to study the asymptotical transition behavior of the network connectivity in terms of the relation of  $c$  and  $k$  and other parameters such as the number  $n$  of nodes, the normalized transmission range  $R_T$ .

The remaining main challenge we will address here is to *statically* assign channels to the NICs of each node such that the resulting wireless network has certain properties, *e.g.*, being connected while using the least number of channels. At a first glance, the problem of assigning channels to wireless nodes appears to be a graph-coloring problem. However a standard node-coloring problem fails to assign channels correctly because it does not capture the constraint where two adjacent communicating nodes of one link need to be assigned a common color (channel). In addition, an edge-coloring formulation also fails to capture the constraint that only a limited number of colors can be incident onto a node due to the limited number of NICs for each node. Furthermore, both formulations fail to capture the property that the resulting network must be connected.

For simplicity, we first assume that we had fixed channels. Notice that, in practice, we may have flexible channels. Furthermore, a node can merge several consecutive channels into one big channel and thus saves the network interface cards it needs. Actually, we can prove that an available flexible channel assignment only increases the throughput of the network, but it will not determine whether there is a channel assignment that results in a connected network.

**Channel Assignment for Connectivity:** We first formally define the channel assignment problem when only network connectivity is required. Remember that throughout this paper, we only consider *static* channel assignment, *i.e.*, each NIC will select one and only one channel from its operable channels to operate on. We notice that in practice dynamic channel switching is possible for some networking cards. Dynamic channel switching clearly can potentially improve the network throughput performance. However, this is not without a price: the hardware needs to be specially designed, and the channel switching does have some overhead such as delay and synchronization between transmitting nodes and receiving nodes where they need to switch to the same channel almost at the same time period. In light of these possible overheads, we mainly consider the static channel assignment where each radio of every wireless node will operate on a pre-determined channel. The channel assignment problem is to determine which channel to operate exactly. Let  $\mathcal{A}(v_i, k, f_j) \in \{0, 1\}$  be the channel allocation function that denotes whether the

$k^{\text{th}}$  NIC of node  $v_i$  selects to operate on channel  $f_i$  or not. Obviously,  $\mathcal{A}(v_i, k, f_j) = 0$  if  $f_j \notin F(v_i, k)$ ;  $\mathcal{A}(v_i, k, f_{j_1}) + \mathcal{A}(v_i, k, f_{j_2}) \leq 1$  since one NIC of a node can only operate on one channel under static channel assignment. When channel assignment is given, we use  $\mathcal{A}(v_i) = \{f_j \mid \mathcal{A}(v_i, k, f_{j_1}) = 1 \text{ for some NIC}\}$  to denote the channels selected by some NICs of node  $v_i$ .

We define the communication graph  $G_{\mathcal{A}} = (V, E')$  induced by the channel assignment method  $\mathcal{A}$  as follows:  $E'$  contains a link  $e = (u, v)$  only if  $e \in E$  and  $\mathcal{A}(u) \cap \mathcal{A}(v) \neq \emptyset$ . We say that a channel assignment  $\mathcal{A}$  is *valid* if the communication graph  $G_{\mathcal{A}}$  is a connected graph. The total channels used by all nodes under  $\mathcal{A}$  is denoted as  $F_{\mathcal{A}} = \bigcup_{u \in V} \mathcal{A}(u)$ . In this paper, we first seek a channel assignment that produces a connected network while using as few channels as possible.

**Channel Assignment for Throughput:** Although network connectivity is an important property of a network, network throughput is a more meaningful criterion to optimize. However, a direct optimization of network throughput involves many aspects of the networking such as traffic prediction, routing, link scheduling and so on, which has drawn a significant amount of research interests recently. A full study of this problem in multi-hop multi-channel multi-radio is out of the scope of this paper. Here we concentrate on assigning channels to nodes such that the resulting communication graph permits the maximum number of simultaneous transmissions, *i.e.*, the size of the maximum independent set in the conflict graph (to be defined later). Two links can transmit at the same time if they will not cause interference with each other. Notice that there is a distinction between the network throughput and the maximum number of simultaneous transmissions permitted in a network. To study the network throughput under a certain channel assignment, we need to specify the interference model to be used. Here we will adopt several widely used interference models as follows.

One of the main obstacles in designing wireless network protocols is the signal interference. A number of interference models have been proposed in literature, such as the protocol interference model [6], the RTS/CTS interference model, and the signal-to-interference-and-noise ratio model, etc. For example, according to IEEE 802.11, a transmitter sends a RTS frame to intended receivers before sending a data frame. If the intended receiver is available to receive data frame, it will respond with a CTS frame to the sender. For every transmitter-receiver pair, all wireless nodes within the interference range of either the transmitter or the receiver will refrain from transmitting. In other words, for two simultaneous transmitting links  $xy$  and  $uv$ ,  $x$  and  $y$  cannot be inside the interference region of either  $u$  or  $v$ , and vice versa. The protocol model (PrIM) was first proposed in [6]. In this model, a transmission from node  $v_i$  (transmitter) to node  $v_j$  (receiver) is successful if and only if the intended receiver  $v_j$  is sufficiently apart from any other transmitter transmitting simultaneously, *i.e.*,  $\|v_k - v_j\| \geq (1 + \eta)\|v_i - v_j\|$ , where  $v_k \neq v_i$  and constant  $\eta > 0$ . Similar with PrIM, Wang *et al.* [24] recently proposed the fixed transmission power protocol interference model (fPrIM) where the transmission from  $v_i$  to  $v_j$  is successful only if  $v_j$

is not within the interference range of any other transmitter  $v_k$  transmitting in the same time slot using the same channel, that is  $\|v_k - v_j\| > R_I(k)$ . Another interference model is the Transmitter Interference Model (TxIM) [27]. In this model, transmission from node  $v_i$  is successful if and only if, for any other transmitter  $v_k$  transmitting in the same time slot using the same channel,  $\|v_i - v_k\| > (1 + \eta)(R_T(i) + R_T(k))$ . Here,  $\eta$  is a system parameter. The physical interference model uses the signal-to-interference-and-noise ratio (SINR) to describe the aggregate interference in the network. In this model, a transmission from node  $v_i$  to node  $v_j$  is successful iff the SINR at the receiver is at least the minimum SINR threshold required.

The channel assignment and link scheduling is closely related to the graph coloring of the conflict graph modeling the wireless networks. Conflict graph modeling has been widely used to model interference in wireless mesh network, which is defined as follows. Consider a graph  $G = (V, E)$  where  $V$  is the set of wireless devices and  $E$  is the set of communication links. The conflict graph  $G' = (V', E')$  corresponding to  $G$  has vertices  $V'$  corresponding to the links in  $E$ , which means that if there is a link  $v_i v_j \in E$ , there will be a vertex  $v_{i,j}$  in  $G'$ . Conflict graph  $G'$  has an edge between two vertices in  $G'$  iff the corresponding links in  $E$  denoted by these two vertices in  $G'$  conflict with each other, i.e., they cannot transmit at the same time slot using the same channel. Specifically, if two links  $v_i v_j$  and  $v_p v_q$  in graph  $G$  conflict with each other, there will be a link  $v_{i,j} v_{p,q}$  in  $G'$  between two vertices  $v_{i,j}$  and  $v_{p,q}$ . Notice that there will be a different conflict graph  $G'$  under constraints of different interference models for the same node set  $V$ , as well as a different channel assignment scheme. Take a simple example, shown in Figure 1, with four wireless

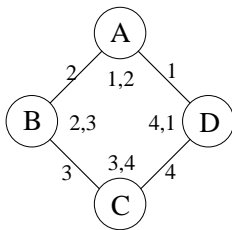


Fig. 1. A simple network represented by a graph  $G$  with four nodes.

nodes,  $V = \{A, B, C, D\}$ , each of them has at least two available NICs. We assume that node  $A$  and  $C$  are not within the interference range of each other, nor are nodes  $B$  and  $D$ . Figure 2(a) shows the conflict graph  $G'$  under the RTS/CTS model without considering the channels and the radios of all nodes. Assume node  $A$  could operate on channels  $\{1, 2\}$ , node  $B$  could operate on channels  $\{2, 3\}$ ,  $C$  operates on channels  $\{3, 4\}$ , and  $D$  operates on channels  $\{1, 4\}$ . When each node has only one NIC, then clearly both  $A$  and  $B$  should use channel 2 to communicate with each other; similarly  $C$  and  $D$  should use channel 4 to communicate with each other. In this case, links  $AB$  and  $AD$  cannot be activated at the same time. Figure 2 (b) shows this phenomenon of conflict when channel availability is taken into account. Figure 2 (c) shows that all 4 links can be activated at the same time if every node has 2 NICs.

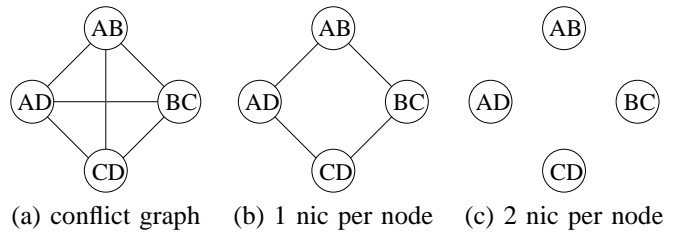


Fig. 2. The corresponding conflict graphs with RTS/CTS interference model.

### III. RELATED WORK

As we know, traditional multi-hop wireless networks are comprised of single radio components, and using multi-radio and multi-channel could efficiently enhance the throughput of the whole wireless network by exploiting the multiple channels and channel reuse opportunities. However, simply using multi-radio and multi-channel without an efficient channel assignment algorithm cannot effectively exploit the increasing bandwidth available, even decrease the network performance due to channel switching overhead and system management overhead. As noted in [15], [19], [22], there are a number of common issues concerning multi-radio, multi-channel, multi-hop wireless networks including network connectivity, sharing channels, switching channels dynamically and network topology.

Concerning the subject of multi-channel multi-hop wireless networks, some researchers have proposed several approaches. For example, in [8], [14], Jain *et al.* proposed a Carrier-Sense Multiple Access (CSMA) protocol based on a modification of IEEE 802.11 for multi-hop wireless networks that uses multiple channels and a dynamic channel selection method. Their main idea was to divide the available bandwidth into  $N$  channels and the transmitting station selects the appropriate channel to transmit packets based on the interference power measurements on all available channels. So *et al.* [20], [21] proposed a MAC protocol requiring only one transceiver per host for ad hoc wireless networks. This MAC protocol enables hosts to utilize multiple channels by switching channels dynamically, thus increasing throughput of wireless networks.

Unlike those approaches above which need to modify the IEEE 802.11, Bahl *et al.* present a link-layer protocol called Slotted Seeded Channel Hopping (SSCH) in [16]. SSCH increases the capacity of an IEEE 802.11 network by utilizing frequency diversity and each node in the wireless network uses SSCH to decrease interference, thus improving the capacity of wireless networks. Das *et al.* [4] present two mixed integer linear programming models for solving the fixed channel assignment problem with multiple radios through finding fixed channel assignment strategies to maximize the number of simultaneous bidirectional links.

There are other examples of other research concerning using multi-NIC for each node, include [17] and [23]. The methods adopted in [17] require each node to have as many NICs as its neighbors have, and also require a sufficiently large number of available channels. In [23], authors assume that there is an *a priori and identical* channel assignment to the NICs of each node, and the channel assignment for all wireless node is the

same: binding #1 NIC with channel 1, #2 NIC with channel 2 and so on. However, as we know, such requirements are unrealistic because the number of NICs each wireless node has may be different and some NICs could only operate on a fixed subset of channels due to the hardware constraints, such as 802.11X series NICs could operate on 11 channels. In this paper, we assume that different wireless nodes may have a different number of NICs, and that the types of NICs do not need to be uniform. Furthermore, the actual available channels for different wireless node could be different, which is more general than previous studies and more close to practice.

Other research such as [2], committed a joint channel assignment scheme for throughput optimization in multi-radio wireless mesh networks. However, it assumes that all nodes have the same transmission range and only considered the static channel assignment situation. In other words, they assume that for each node  $v$ , the number of available channels for it is no more than the number of NICs  $I(v)$ . However, as we know, in practice, not only could the transmission ranges of wireless nodes be different, but also nodes could adjust the channels they operate on dynamically.

#### IV. TRANSITION PHENOMENA

Recent work [7], [12] has shown that, when  $n$  nodes are randomly deployed in a certain region, there is a critical transmission power  $r_n$  required to ensure with high probability that two wireless nodes in the network could communicate with each other through one- or multi-hop paths if all nodes have fixed power  $r_n$ . In other words, such critical transmission power is also the threshold of keeping the whole network connected, if the area of the plane and the number of the nodes in the networks are fixed. Generally, it was known that, given  $n$  nodes of set  $V$  randomly deployed in a unit area region (such as a square or a disk) and every node with transmission range  $r_n$ , the unit disk graph  $G(V, r_n)$  formed by  $V$  (which has a link  $uv$  iff  $\|u - v\| \leq r_n$ ) is connected with high probability if  $\pi n r_n^2 \geq \log n$ , otherwise it is disconnected with high probability.

For multi-radio, multi-channel, multi-hop wireless networks, several networking parameters will affect the network connectivity, for example, the transmission range of wireless nodes, the number of radios available for every node, the number of channels that can be used by each radio. Traditional results on critical transmission range for connectivity assumed that every wireless node only has one NIC and there is only one channel to choose. In a wireless network with multi-radios, we assume that channels which can be operated by a radio of a specific wireless node are randomly chosen from a set of channels. Then we would like to study whether there is a critical density of channels for the network connectivity. Clearly, we first need the transmission range  $r_n$  to satisfy the requirement  $\pi n r_n^2 \geq \log n$ .

In multi-radio multi-channel networks, we generally assume that there are  $k$  channels  $\mathcal{F}$  available globally and each radio will be able to operate on  $s$  channels randomly selected from  $\mathcal{F}$ . Notice that when two end-nodes of a link  $uv$  in  $G(V, r_n)$  do not have a common operable channel, then this link  $uv$

will not be operable by  $u$  and  $v$  although nodes  $u$  and  $v$  are physically within the transmission range of each other. We define a network  $G(V, r_n, s, k)$  as the set of links  $uv$  where  $\|u - v\| \leq r_n$  and  $u$  and  $v$  have a common channel. Clearly, if  $s > \frac{k}{2}$ , then it is guaranteed that two end nodes of a link  $uv$  in  $G(V, r_n)$  will have a common channel to communicate, *i.e.*,  $G(V, r_n, s, k)$  is connected as long as  $G(V, r_n)$  is connected. We want to know the critical value for  $s$  (with fixed  $r_n$  and  $k$ ) that will allow the network  $G(V, r_n, s, k)$  to remain connected.

Obviously, a link  $uv$  remains in  $G(V, r_n, s, k)$  if they have a common channel, whose probability is

$$P_{k,s} = 1 - \frac{\binom{k}{s} \binom{k-s}{s}}{\binom{k}{s} \binom{k}{s}}$$

if every node only has one radio. When every node has same set of  $R$  distinctive radios, the probability that two nodes will have a common channel among any of these  $R$  radios is

$$P_{k,s,R} = 1 - \left( \frac{\binom{k}{s} \binom{k-s}{s}}{\binom{k}{s} \binom{k}{s}} \right)^R$$

Consequently, every link in  $G(V, r_n)$  will survive in  $G(V, r_n, s, k)$  with a probability  $P_{k,s,R}$ , *i.e.*, the network  $G(V, r_n, s, k)$  can be modeled as a Bernoulli model. From [26], we know that the network  $G(V, r_n, s, k)$  is connected with high probability only if

$$\pi \cdot P_{k,s,R} \cdot n \cdot r_n^2 \geq \log n.$$

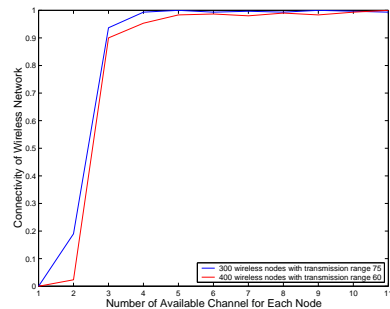


Fig. 3. Transition phenomena in wireless networks.

The simulation results of transition phenomena of network connectivity are reported in Figure 3. Here, the  $x$ -coordinate indicates the number of available channels on which each node could operate during a period of time, and  $y$ -coordinate denotes the connectivity of the whole wireless network. In this simulation, we randomly generate 300 and 400 wireless nodes respectively in a  $500 \times 500$  unit region and the difference between these two cases is that the transmission range of the former is 75 units and 50 units of the latter. Typically, a unit represents about one meter here. We assume the maximum number of available channels on which the wireless nodes could operate is 11, in practice, this number maybe different, for example, IEEE 802.11b specifies 11 channels in the 2.4 GHz spectrum, 3 of which are orthogonal, and IEEE 802.11a specifies 13 orthogonal channels in the 5 GHz. We vary the number of available channels for wireless nodes from 1 to 11, and the results indicate that the connectivity of a wireless

network increases sharply from 2 to 4 available channels. The whole wireless network is connected almost 100 percent provided that the available channels are equal to or greater than 4.

## V. OUR APPROACHES

In the previous section, we showed that for a network to be connected, we need at least a certain number of channels for each node if the channels are randomly available. In this section, we are ready to present our approaches for static channel assignment while achieving connected networks, and/or networks permitting largest number of simultaneous transmissions.

### A. Complexity Result

First of all, given a wireless network and the radio and channel availability constraints of all wireless nodes, we show that it is NP-complete to decide whether we have a channel assignment that results in a connected network without channel switching.

*Theorem 1:* It is NP-complete to decide whether we have a channel assignment that results in a connected network without channel switching (*i.e.*, the number of assigned channels to each node is no greater than its number of NICs).

*Proof:* Clearly the problem is NP since we can check whether a given assignment is valid in polynomial time. We then show that it is NP-hard from the induction of a NP-complete problem finding a minimum-degree spanning tree.

It is known that it is NP-complete to decide whether a given graph  $G$  has a spanning tree whose maximum node degree is at most a given integer  $D$  [5]. Given the graph  $G = (V, E)$  and integer  $D$ , we construct a channel assignment problem for a network  $G'$  as follows. The network  $G' = (V', E')$  is the same as  $G$ , and each wireless node of  $V'$  will have  $D$  radios. There are  $m$  total channels, where each channel corresponds to an edge in the graph  $G$ . The set of channels on which a wireless node can operate is the set of channels corresponding to all incident edges of this node in  $G$ . We then show that graph  $G$  has a spanning tree  $T$  with a maximum degree  $D$  iff there is a channel assignment for  $G'$  that results in a connected network without channel switching.

First of all, if such a tree  $T$  does exist, then our channel assignment for the network  $G'$  will work as follows: a node  $u$  is assigned a channel represented by the edge  $e$  if  $e$  is an edge incident on  $u$  in the tree  $T$ . Clearly, the total number of channels assigned to every node  $u$  is at most  $D$ , and the network induced by the channel assignment is obviously connected since  $T$  will be a subgraph of the induced network.

Secondly, assume there is a channel assignment that induces a connected network. Since each node has at most  $D$  radios, the number of channels assigned to every node is at most  $D$  and thus the induced network has the maximum degree  $D$ . Consequently, any spanning of the induced network is a spanning tree for  $G$  whose maximum degree is at most  $D$ . This finishes the proof. ■

In the remainder of this section, we propose several novel methods to assign channels to every node in order to achieve

a certain network property. Notice that given  $n$  wireless nodes randomly distributed in a region, whether those nodes could form a connected network or not has been determined when the attributes of nodes are given, such as the transmission range, interference range, available channels and NICs of nodes.

### B. Backbone Based Approach

Assume there are  $n$  wireless nodes randomly distributed in a region and each node  $v_i$  has  $I(v_i)$  NICs. The transmission range of  $v_i$  is  $R_T(i)$ . Every node  $v_i$  has some adjacent neighbor nodes, denoted by  $N(v_i)$ . Here if  $v_j \in N(v_i)$ , then  $v_j$  should be within the transmission range of  $v_i$  and node  $v_i$  and  $v_j$  should have one or more common channels, that is  $F(i) \cap F(j) \neq \emptyset$ . Notice that,  $v_j \in N(v_i)$  does not mean  $v_i$  also belongs to  $N(v_j)$  due to the different transmission range of  $v_i$  and  $v_j$ . Notice that  $N(v_i)$  could be empty in some special case, and the network  $G$  is then not connected. We always assume that the network is connected by having enough channels to wireless nodes. On the other hand, assume we have enough channels and we would like to minimize the number of channels needed to guarantee the whole network connected.

Our first method is a Backbone Based Approach (BBA). Obviously, the centralized method will get the best channel assignment performance. However, a centralized approach, letting some node collect information of all other nodes in the network, is almost impossible due to the low memory and potentially high mobility of wireless nodes. Even if possible, it is unwise because the whole network would be full of messages regarding the information of wireless nodes, which will sharply decrease the throughput of networks. The main idea of our backbone based approach is to let every node only record some necessary information of some neighboring nodes, typically at most 3 hops away. Our backbone based approach will partition the wireless nodes into clusters: a clusterhead in each cluster will be in charge of the channel assignment for all nodes in the cluster; the clusters are connected via gateway nodes which will ensure that the resulting network is connected. The benefits of using backbone based approach are that (1) the majority channel assignment will be restricted to a small local neighborhood, thus, it is possible to find the optimal assignment for this small local neighborhood; (2) the backbone is sparse and already connected, thus making it much easier to have a channel assignment that results in a connected network. Through the backbone, a node  $v_i$  could communicate with any other node  $v_j$  in the network through one or multi-hop communication.

In BBA, we divide all nodes into three different type sets: *Dominators*, *Dominatees* and *Connectors*. A typical method for building a dominating set typically works as follows. Initially all nodes are marked as white (whose status is not determined yet). A white node claims itself to be a dominator if it has the largest ID (or some other ranking methods such as the number of radios or the node degree) among all of its neighbors, then it will broadcast a message *IamDominator* to its one-hop neighbors, and mark itself *Dominator*. Here we assume that each node knows the IDs of its one-hop neighbors.

When a white node receives an *IamDominator* message, it will mark itself as a *Dominatee* and then broadcast *IamDominatee* to its one-hop neighbors. Through the procedure above, the set of dominators is actually a maximal independent set in which there are no adjacent nodes (*i.e.*, two nodes could communicate directly in one-hop). For simplicity, a dominatee node will choose only one dominator from possibly several neighboring dominators. If the shortest path connecting a dominator  $v_i$  and a dominator  $v_j$  is  $k$ -hops, we say  $v_j$  is a  $k$ -hop dominator-neighbor of  $v_i$ , and vice versa. Typically, in this paper, we need  $k$  to be 2 or 3.

Here we also define another criterion called *channel-degree* for selecting dominators. Assume that a node  $v_i$  has  $k$  one-hop neighbors  $N(v_i)$  and for a neighboring node  $v_j$ , the number of common channels between  $v_i$  and  $v_j$  is  $c_{ij}$ . Then the channel-degree of node  $v_i$  is  $\sum_{v_j \in N(v_i)} c_{ij}$ . When channel-degree is used for building a maximal independent set, we select a white node to MIS if it has the largest channel-degree among all its white neighbors.

After getting the dominator set and dominatee set, the next step is to choose some nodes from dominatee set as gateway nodes (also called connectors) to connect all dominator nodes and thus the backbone of whole network. There are several different centralized approaches to find connectors.

One way is to let the dominatees which are dominated by most dominators have priority to become connectors. First, all dominatees are in the candidate set of connectors. We pick the dominatee with the most dominators from the candidate set and make it a connector. Then we delete it from candidate set. Second, we find the next dominatee with the most dominators from the remaining candidates, if this dominatee could connect at least one unconnected dominator or connect two different trees composed of dominators and connectors, it claims itself to be a connector; otherwise, we delete it from candidate set. We find the next candidate node and repeat the above procedure until all dominators are connected completely. Here, we assume that every dominatee node knows how many and which dominators could dominate itself.

Another way to connect dominators is known as Steiner tree algorithm. This straightforward approach gives the approximation ratio  $c \cdot (H(\Delta) + 1)$ , where  $c$  is the approximation ratio for the unweighted Steiner tree problem. Currently, the best ratio for  $c$  is  $1 + \frac{2}{\ln 3} \approx 1.55$ , by Robins and Zelikovsky [18]. However, all of the approaches above are centralized and this is not practical.

We use two different approaches to elect connectors from dominatees and construct the backbone of a wireless network with dominators.

**Locally Selected Connectors:** One efficient approach is that for each pair of dominators  $v_i$  and  $v_j$  that are at most 3 hops away from each other, we find one shortest path connecting them and add all links of this path into the backbone. It has been proved in [3], [25] that the size of the backbone constructed as above is at most a small constant factor of the optimum when the network is modeled by a unit disk graph. Notice that it is possible that there may be multiple paths selected to connect two dominators  $u$  and  $v$ , thus making the backbone more robust. See Figure 4 for an illustration of a

wireless backbone.

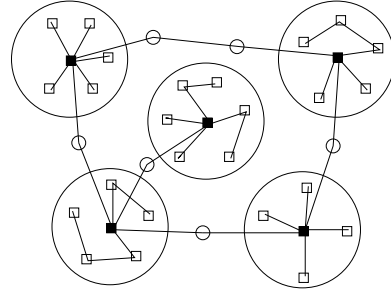


Fig. 4. Backbone of a wireless network.

**Minimum Spanning Tree (MST) Backbone:** The above method can select a small number of gateway nodes; however, if the backbone is too dense, it may not be good for our channel assignment. Notice that we need to result in a connected network, thus, more links in the backbone means that our channel assignment should satisfy more constraints (a link in the backbone poses a constraint such that the end-nodes of this link should be assigned at least one common channel). In light of this, we propose to use the minimum spanning tree to build the backbone, which is the sparsest backbone we could get. For every pair of dominators  $u$  and  $v$  that are 2 or 3 hops away, we assume that there is a virtual link connecting them with the cost defined as the Euclidean distance  $\|u - v\|$  between them, or  $\frac{1}{F(u)+F(v)}$ . The intuition behind  $\frac{1}{F(u)+F(v)}$  is that we want to connect pairs of dominators who have more channel availability, which will make it easier to get a connected network without channel switching. For distributed implementation, we can also apply a local minimum spanning tree structure proposed in [10], [11].

We then study how to assign channels to dominatees, especially all nodes dominated by one specific dominator node. One simple approach could just use the star structure: every dominatee node is connected to the dominator node and we want to assign channels to them such that these links between the dominator and the dominatee nodes will have common channels. This simple approach clearly will put a heavy burden on the dominator node since its channel assignment needs to satisfy the connections to *all* its dominatees, which may be impossible. We adopt another simple approach as follows. We build some sort of minimum spanning tree to connect the dominator and all its dominatees (excluding the connector nodes). The selection of the link cost will clearly affect the performance. If Euclidean distance is used, we know that every node has at most 5 neighbors in the minimum spanning tree; therefore, it only needs to ensure that it has common channels with these neighbors. Algorithm 1 illustrates our approach of constructing local minimum spanning tree based on the channel availability of different nodes.

### C. Spanning Tree Based Approach (STBA)

In this subsection, we propose another minimum spanning tree method to find a structure as a base of our channel assignment. We will also compare the performances of MST-based structure and cluster-based structure. Assume there are

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**Algorithm 1** Spanning Tree for Local Neighborhood

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**Input:** Assume there are  $n$  wireless nodes  $V = \{v_1, v_2, \dots, v_n\}$  in the local area  $\mathcal{Z}$ , including one dominator and a number of dominatees, and  $E$  is the set of all possible links (edges) whose start points and end points are in  $V$ . Let  $E$  be all links formed by nodes  $V$  where  $v_i v_j \in E$  iff they are within the transmission range of each other and share a common operable channel.

**Output:** A spanning tree  $G = (V, E')$ .

- 1:  $V' = \emptyset; E' = \emptyset$
  - 2: Collect all available channels which could be used by all links in  $E$ , and save these channels into the set  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$  by decreasing order of usage, that is,  $c_1$  is the channel which could be operated most by all links. Set  $i = 1$ .
  - 3: **while**  $i \leq k$  **do**
  - 4: Find all links  $v_p v_q$  that can operate on channel  $c_i$ , here  $v_p$  and  $v_q$  are from different connected components of graph  $G = (V, E')$ . The set of such links is denoted as  $E_i$ .
  - 5: **for** each link  $v_p v_q \in E_i$  **do**
  - 6: Add  $v_p v_q$  to  $E'$  if  $v_p$  and  $v_q$  are from different connected components of  $G = (V, E')$ , and update graph  $G$  accordingly.
  - 7: Remove link  $v_p v_q$  from  $E_i$ .
  - 8: **end for**
  - 9: Update  $i = i + 1$ .
  - 10: **end while**
- 

$n$  wireless nodes in a region  $\mathcal{Z}$ , and  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$  is the set of channels which could be operated on by all  $n$  nodes. Assume each node must have at least one NIC which could operate on one of channels in  $\mathcal{C}$ . We adopt two ways to compute the weight of links. One is to use the Euclidean distance  $\|v_i - v_j\|$  as the weight of link  $v_i v_j$ . We use Prim's algorithm to implement the Minimum Spanning Tree. The other one is to use fewest number of channels to connect all nodes, which means when we choose one link from all candidate links, we always pick up the link that will lead to a *minimum* increase of the number of channels already used by all selected links. Algorithm 2 presents our method that constructs a spanning tree for channel assignment.

#### D. Assign Channels

After the structures have been constructed, we now are ready to show how to assign channels to radios of every node such that the resulting network will be connected. Given a structure  $H$  (typically a connected spanning tree), we propose two different approaches to assign channels to links of the structure  $H$ . The first approach will always find a channel, say  $c$ , that can be used by the greatest number of *unassigned* links in  $H$  and then assign  $c$  to all appropriate links in  $H$  (*i.e.* the end nodes of these links); these links are called assigned. Originally, all links in  $H$  are unassigned. Notice that Algorithm 2 also produces such an assignment at the time of constructing the spanning tree. Observe that such an

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**Algorithm 2** Minimum Incremental Spanning Tree Structure

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**Input:**  $G = (V, E)$

**Output:** Spanning Tree  $G_{ST} = (V', E')$ .

- 1:  $V' = \emptyset; E' = \emptyset$  Let  $c_1$  be the channel that could be operated by the most links.
  - 2: Pick a random link  $v_1 v_2$  with a common channel  $c_1$ . Add  $v_1$  and  $v_2$  to  $V'$ . Let  $\mathcal{C}$  be the set of channels assigned to nodes in  $V'$  so far.
  - 3: **while**  $|E'| < n - 1$  **do**
  - 4: Pick a link  $v_i v_j$  such that either  $v_i \notin V'$  or  $v_j \notin V'$  or both, and there is a common channel between  $v_i$  and  $v_j$  using channels from  $\mathcal{C}$ . If no such link exists, find a channel  $c$  that is used by the most links with at least one end-nodes from  $V - V'$ . Then pick a link  $v_i v_j$  such that either  $v_i \notin V'$  or  $v_j \notin V'$  or both, and it has a common channel  $c$ . Let  $\mathcal{C} = \mathcal{C} \cup \{c\}$ . Add link  $v_i v_j$  to  $E'$  and  $V' = V' \cup \{v_i, v_j\}$ .
  - 5: **end while**
- 

assignment may not always be valid: sometimes the number of assigned channels to a node may be more than the number of the NICs it has.

When the structure  $H$  is a tree, we actually can perform the channel assignment as follows. Assume that the tree is rooted. Pick a leaf node  $v$  and assign  $v$  and its parent node  $u$  a channel  $c$  that is mostly used by links incident on  $u$ . Remove node  $v$  and its incident link from the tree  $H$ . Repeat the above steps until  $H$  becomes empty.

## VI. PERFORMANCE EVALUATION

In this section, we present our simulation results that evaluate our algorithms to study the performance gain of using multi-radio, multi-channel and multi-hop for wireless mesh networks. Besides the transition phenomenon which has been illustrated in IV, we demonstrate the performance of both the Backbone Based Approach and the Spanning Tree Based approach. In our simulation, we assume that each wireless node's NIC could operate on all 11 channels, and the wireless node will not dynamically switch the channel during a certain time period. We assume there are 300 wireless nodes randomly distributed in a  $500 \times 500$  units (meters) area and the transmission range for each wireless node is 75 units.

We first compare the performance of two channel assignments obtained by our Backbone Based Approach and Spanning Tree Based Approach respectively. Figure 5 shows the results of the minimum number of channels required by BBA and STBA to keep the whole network connected. We vary the number of available NICs from 5 to 11, and randomly bind each NIC with one of 11 channels. Through the two curves we know that BBA needs fewer channels to make the wireless network connected under the same condition.

We then study the number of simultaneous transmission links that can be supported by the resulting network from a given channel assignment method. To some extent, the more simultaneously transmission links exist, the more throughput

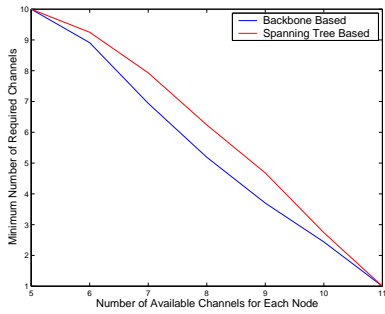


Fig. 5. Comparison result between BBA and STBA for number of needed channels.

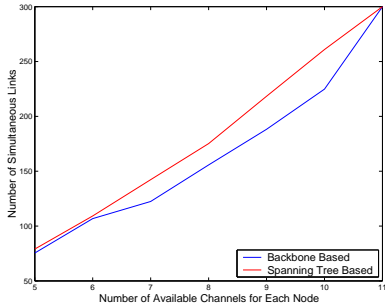


Fig. 6. Comparison result between BBA and STBA on the number of simultaneous transmitting links.

of the whole wireless network will be obtained. As can be seen from Figure 6, the maximum number of links which could transmit data simultaneously in the resulting wireless network obtained by our two approaches have a little difference too. Clearly, the Spanning Tree Based Approach performance better than the Backbone Base Approach does on this point. Here, we assume for each node  $v_i$ ,  $R_I(i) = 1.5 \times R_T(i)$ , and the interference range of a link  $e(v_i v_j)$  is  $R_I(e) = R_I(i) \cup R_I(j)$ . We adopt the RTS/CTS interference model in our simulation and thus if a link  $e'$  has any one end in the  $R_I(e)$ , and if  $e'$  and  $e$  are assigned the same channel in channel assignment method, they will interfere with each other. We also measure

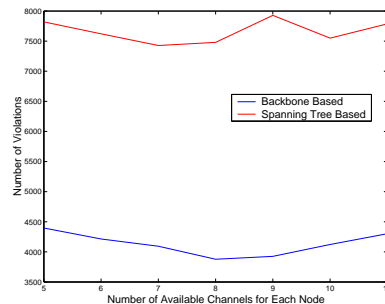


Fig. 7. Total link interferences among all links in the network.

the number of total interference among all links. Observe that the number of total interference directly influences other issues like link scheduling, and thus influences the throughput of whole network. Figure 7 presents the result of our simulation concerning this total interference number for all links of connected wireless network. We found that the backbone based

channel assignment method produces the least amount of total link interferences.

Notice that in a channel assignment produced by some of the above approaches we cannot simultaneously *guarantee* that the resulting network is connected and the number of assigned channels to a node is not greater than its number of NICs. Remember that it is NP-hard to decide whether we can achieve the above for an arbitrary network. If a node has more assigned channels than the number of NICs it has, then clearly this node needs to perform channel switching during the routing, and this causes not only delay but also some other overhead in routing. We then study the number of NIC violations the network will have to get a connected network under a certain channel assignment method. Here the NIC violation of a node is defined as the number of assigned channels to this node minus the number of NICs this node has if assigned channels are larger than the number of NICs; otherwise, it is 0. We then conducted extensive simulations to study the expected number of violations in a channel assignment. In our simulations, we randomly generate 300 nodes and the transmission range of each node is 75 units. The number of NICs for each node is 3 and every NIC could operate on 3 of 11 channels. We randomly generate 300 network instances and then produce data for Table I. Notice that when the number

TABLE I  
NUMBER OF AVERAGE NIC VIOLATIONS FOR A CHANNEL ASSIGNMENT.

# of Channels	6	7	8	9	10	11
CDS	2.2608	2.2556	1.3622	0.422	0.02	0
MST	0	0.1111	0.0165	0.0035	0	0

of available channels is 6 or 7, the connectivity of the original network is very bad (about 5 network instances out of 300 samples are connected in our simulations). When the number of NICs comes to 8, 9, 10, the network is connected with high probability. The number reported in Table I is the average of the performances of these network instances where the original network is connected. Based on our simulations, we found that MST often will have fewer NIC violations: most times it will only have about 1 channel switching needed in the whole network. On the other hand, when the original network is connected, the connected dominating set (CDS) also only has an insignificant number of NIC violations. These simulations confirm that our backbone based channel assignment method indeed will give us a connected network without channel switching with high probability.

We also conducted extensive simulations to study two different channel assignment methods and found that the channel assignment assigning the most widely used channels first always has the best performances compared with the method specifically designed for the tree.

## VII. CONCLUSION

In this paper, we studied the *static* channel assignment problem by statically mapping a channel to each radio of the nodes. We showed that it is NP-complete to find a channel assignment such that the resulting network is connected. We then gave several novel methods that only use a small

number of channels while resulting in a connected network. We presented a novel method to assign channels such that the network will support a large number of simultaneous transmissions (thus possibly increasing the network throughput). We also theoretically analyzed the expected number of channels that a node needs to be able to operate so that it can result in a connected network with high probability. We conducted extensive simulations to study the performances of our algorithms. One important future research is to design a centralized and a message efficient distributed algorithm such that they have a theoretically proven worst case performance guarantee.

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