Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, January 13, 2018

12:00 - 4:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

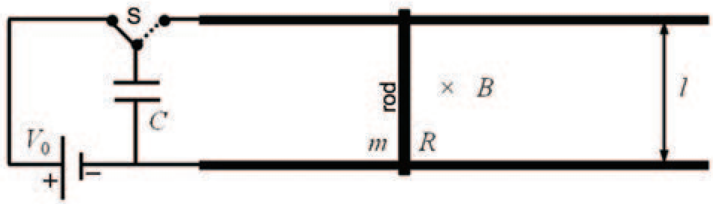
Physical Constants

Speed of light in vacuum	c	=	2.998×10^{8} m/s
Planck's constant	h	=	6.626×10^{-34} J· s
	$\hbar$	=	$h/2\pi$
		=	1.055×10^{-34} J·s
		=	6.582×10^{-16} eV·s
Permeability constant	μ_o	=	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	=	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	=	$\frac{e^2}{4\pi\epsilon_o \hbar c}$
		=	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	=	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	=	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	=	1.381×10^{-23} J/K
		=	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	=	0.0258 eV
Universal gas constant	R	=	8.314 J/mole·K
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	=	1.602×10^{-19} C
Electron rest mass	m_e	=	9.109×10^{-31} kg
		=	0.5110 MeV/c^2
Neutron rest mass	m_n	=	1.675×10^{-27} kg
		=	939.6 MeV/c^2
Proton rest mass	m_p	=	1.672×10^{-27} kg
		=	938.3 MeV/c^2
Deuteron rest mass	m_d	=	3.343×10^{-27} kg
		=	1875.6 MeV/c^2
Atomic mass unit (C^{12} = 12)	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Mass of earth	M_E	=	5.98×10^{24} kg
Radius of earth	R_E	=	6.37×10^{6} m
Mass of sun	M_S	=	1.99×10^{30} kg
Radius of sun	R_S	=	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^2
Atmospheric pressure		=	1.01×10^{5} N/m^2
Radius of earth's orbit		=	1.50×10^{11} m
Radius of moon's orbit		=	3.84×10^{8} m

Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 1:



One end of a horizontal track of width l and negligible resistance is connected to a capacitor of capacitance C charged to voltage V_0 of polarity shown in the figure. The inductance of the assembly is negligible. The system is placed in a homogeneous vertical magnetic field B pointing into the page. A frictionless conducting rod of mass m and resistance R is placed perpendicular onto the track. After the capacitor is fully charged the position of the switch S is changed from the position indicated by the full line to the position indicated by the dotted line, and the rod starts moving.

(a) In which direction does the rod move, and why?

(b) What is the maximum velocity that the rod acquires?

Problem 2:

A charge density ρ_0 is placed at time $t = 0$ in a small region in the interior of a homogeneous charge-neutral material that has electrical conductivity σ.

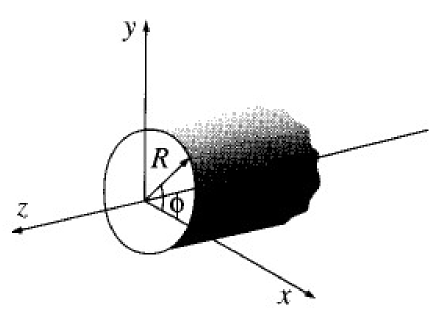
(a) Derive an expression for the time evolution of the charge density in that region, $\rho_c(t)$, with $\rho_c(0) = \rho_0$. Hint: use a continuity equation.

(b) Estimate how long it will take (in seconds) for the charge density to decrease to 1/1000 of its initial value if the material is (i) copper with conductivity $\sigma = 1/(2 \times 10^{-8}\ \Omega m)$ and (ii) quartz with conductivity $\sigma = 1/(10^{16}\ \Omega m)$.

Use $\varepsilon_0 = 8.85 \times 10^{12}\ \mathrm{C^2/Nm^2}$.

Problem 3:

A surface of an infinite cylinder of radius R is charged with the charge density$\sigma(\phi) = a\sin(5\phi)$. Find the electrostatic potential inside and outside of the cylinder.



Problem 4:

A one-dimensional particle of mass m and energy E is incident on the δ-function potential $V(x) = V_0\delta(x)$.

(a) Find the reflection and transmission coefficients.

(b) Find the phase shift δ of the transmitted wave, and the difference

$$\delta(E \to \infty) - \delta(E \to 0).$$

Problem 5:

Consider an electron constrained to move in the xy plane under the influence of a uniform magnetic field of magnitude B oriented in the $+\hat{z}$ direction. The Hamiltonian for this electron is

$$\mathbf{H} = \frac{1}{2m}\left(\left(\mathbf{p}_x - \frac{e}{c}A_x\right)^2 + \left(\mathbf{p}_y - \frac{e}{c}A_y\right)^2\right),$$

where m and e are the mass and charge of the electron, and c is the speed of light.

(a) Find a suitable expression for $\vec{A}$ so that $\mathbf{p}_x$ is a constant of motion for the above Hamiltonian.

(b) With this choice for $\vec{A}$, show that the eigenfunctions of $\mathbf{H}$ can be written in the form

$$\Psi(x, y) = e^{\frac{i}{\hbar}p_x x}\Phi(y),$$

where $\Phi(y)$ satisfies the Schrödinger equation for a one-dimensional harmonic oscillator whose equilibrium position is $y = y_0$. Find the effective spring constant k for this oscillator and the shift of the origin y_0 in terms of p_x, B, m, e, c.

(c) Find the energy eigenvalues for this system, and indicate degeneracies.

(d) For the remainder of the problem, suppose we further restrict the particles to live in a square of side length L. Suppose we demand periodic boundary conditions. What are the possible values of p_x?

Problem 6:

Prove the following relation, where $\hat{l}$ is an angular momentum operator:

$$\left[\hat{l}, p_x^2 + p_y^2 + p_z^2\right] = 0.$$

Problem 7:

In a December IIT M.S. Thesis, the production and decay of a supersymmetric single top *squark* $\tilde{t}$ at the Large Hadron Collider was calculated in the process $pp \to \tilde{t} \to t\chi_1^0$, where t is a top quark, and χ_1^0 is a neutralino. Assume the masses of the particles are $m_{\tilde{t}} = 5\,\text{TeV/c}^2$, $m_t = 175\,\text{GeV/c}^2$, and $m_{\chi_1^0} = 200\,\text{GeV/c}^2$.

(a) Find the energy of the top quark if the top *squark* is produced at rest.

(b) The top quark decays via $t \to bW$, where $m_b = 5\,\text{GeV/c}^2$ and $m_W = 80.4\,\text{GeV/c}^2$. What is the maximum energy of the b quark in the final state of $\tilde{t}$ production above?

Problem 8: Observations of quasar 3C-9 suggest that light currently reaching Earth was emitted while the quasar was moving $0.8c$ away from our solar system. If one of the identified spectral lines is at 1200 Åwhen observed from a stationary source, at what wavelength did this line appear in the spectrum of the quasar?