Math 431 – Computational Algebraic Geometry

Course Description (Bulletin): Systems of polynomial equations and ideals in polynomial rings; solution sets of systems of equations and algebraic varieties in affine n-space; effective manipulation of ideals and varieties, algorithms for basic algebraic computations; Groebner bases; applications. Credit may not be granted for both MATH 431 and MATH 530. (3-0-3) (C)

Enrollment: Elective for AM and other majors.

Textbook(s): Cox, Little and O'Shea: Ideals, Varieties and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra.
ISBN 978-1-4419-2257-1 (3rd edition).

Other required material: Use of computer algebra system, such as Macaulay2, Singular, CoCoA, or Sage. All are free/open source.

Prerequisites: MATH 230, MATH 332

Objectives:

1. Students will achieve command of the essentials of computational algebraic geometry and commutative algebra.
2. Students will understand and apply the core definitions and theorems, generating examples as needed, and asking the next natural question.
3. Students will achieve proficiency in constructing proofs, including those using basic polynomial ideal theory, basic algorithms and constructive proofs, basic varieties and existence proofs.
4. Students will achieve proficiency in written and oral communication of proofs and concepts of both pure and applied computational algebraic geometry.
5. Students will become familiar with the major viewpoints and goals of algebraic geometry: ideals, varieties, and algorithms.
6. Students will practice their knowledge of abstract algebra to problems with exercises and applications, through the required use of a computer algebra system, as well as a class project which will consist of reading an extra chapter or a research paper on the topic from the course.

Lecture schedule: 3 50 minute (or 2 75 minute) lectures per week

Course Outline:

Topic	Hours
What is applied algebra? Preliminaries: basic introduction to algebraic structures (fields, rings).	3
Polynomials and affine spaces. Affine varieties and their parametrizations.	6

Ideals in the polynomial ring.	3
Polynomials in one variable, and Introduction to algorithms/pseudocode.	3
Monomial orderings and division algorithm in many variables. Dickson’s Lemma. The Hilbert basis theorem. The ascending chain condition.	8
Groebner bases and their properties. S-pairs. Buchberger's algorithm and first application of Groebner bases.	6
Elimination and extension theorems. Geometry of elimination. Implicitization problem and algorithms for polynomial and rational implicitization. Resultants.	6
Hilbert's NullStellenSatz. Radical ideals, ideal-variety correspondence, radical membership.	3

Note: Some of the last three topics may be covered in less depth depending on time constraints. In some semesters, emphasis may be placed on one of the three final topics more so than the other two, in order to cover it in more depth.

Assessment:
Homework 15-30%
Quizzes/discussion/participation 10%
Mid-term exam 20-25%
Final Exam 20-30%
Project 15-25%

Syllabus prepared by: Sonja Petrović, 2/24/2015 (updated 07/08/2015)