MATH 478 – Numerical Methods for Differential Equations

Course Description from Bulletin: Polynomial interpolation; numerical integration; numerical solution of initial value problems for ordinary differential equations by single and multi-step methods, Runge-Kutta, Predictor-Corrector; numerical solution of boundary value problems for ordinary differential equations by shooting methods, finite differences and spectral methods. (3-0-3)

Enrollment: Elective for AM and other majors

Textbook(s): A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, 2^{nd} Ed., Cambridge University Press (2008), ISBN 0-521-73490-8 (paperback).
D. Kincaid and W. Cheney, *Numerical Analysis: Mathematics of Scientific Computing*, 3rd Ed, Brooks/Cole (2002), ISBN 0-534-38905-8.

Other required material: Matlab

Prerequisites: MATH 350 Introduction to Computational Mathematics or MMAE 350, or consent of the instructor

Objectives:
1. Students will learn the basic polynomial interpolation methods and their use with numerical integration methods.
2. Students will learn the basic numerical methods for solving initial value problems and their characteristic properties.
3. Students will learn the concepts of order, stability, and convergence of a numerical method.
4. Students will learn the basic numerical methods for solving boundary value problems and their characteristic properties.
5. Students will learn how to implement and use these numerical methods in Matlab (or another similar software package).

Lecture schedule: 3 50 minutes (or 2 75 minutes) lectures per week

Course Outline: Hours

1. Mathematical background — 10
    a. Lipschitz continuity
    b. Taylor polynomials and polynomial interpolation, splines
    c. Numerical integration methods
    d. Richardson Extrapolation
    e. Existence and uniqueness theorem for initial value problems
2. Nonlinear algebraic systems — 3
    a. Fixed-point iteration
    b. Newton-Raphson iteration
3. Single step methods for differential equations — 5
    a. Derivation of Euler and Taylor methods, trapezoidal rule, theta method
    b. Order and convergence
4. Multistep methods for differential equations — 7

    a. Derivation of Adams methods, general multistep methods, BDFs
    b. Order and convergence
    c. Dahlquist equivalence theorem
5. Runge-Kutta methods 3
    a. Derivation
    b. General form
6. Stability and Stiff equations 3
    a. Linear stability analysis
    b. Stiffness
    c. A-Stability
7. Error control 4
    a. Adaptive stepsize control
    b. Predictor-Corrector methods
    c. Embedded Runge-Kutta methods
8. Boundary value problems 7
    a. Shooting methods
    b. Finite differences
    c. FFT and spectral method

Assessment:

	Homework	10-30%
	Computer Programs/Project	10-20%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

Syllabus prepared by: Greg Fasshauer and Xiaofan Li
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