MATH 589 – Numerical Methods for Partial Differential Equations

Course Description from Bulletin: The course introduces numerical methods, especially the finite difference method for solving different types of partial differential equations. The main numerical issues such as convergence and stability will be discussed. It also includes an introduction to the finite volume method, finite element method and spectral method. (3-0-3).

Enrollment: Elective for graduate student

Textbook(s): K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, Cambridge, 2nd Edition

Other required material:

Prerequisites: Undergraduate courses in numerical methods (such as Math 350) and in partial differential equations (such as Math 489), or consent of the instructor

Objectives:
1. Students should learn the principles for designing numerical schemes for PDEs, in particular, finite difference schemes.
2. Students should learn to make a connection between the mathematical equations or properties and the corresponding physical meanings.
3. Students should be able to analyze the consistency, stability and convergence of a numerical scheme.
4. Students should know, for each type of PDEs (hyperbolic, parabolic and elliptic), what kind of numerical methods are best suited for and the reasons behind these choices.
5. Students should be able to use a programming language or math software (Matlab, Maple or Mathematica) to implement and test the numerical schemes.

Lecture schedule: Two 75 minute lectures

Course Outline:	Hours
1. Introduction: Some physics behind the PDEs	2
2. Parabolic equations in 1-D	12
a. Explicit and implicit finite difference schemes	
b. Truncation errors and consistency	
c. Stability analysis: matrix method, maximum principle, Fourier analysis, energy method	
d. Maximum principle and convergence	
e. Lax equivalence theorem	
f. Practical/strict stability	
g. General boundary conditions and nonlinear problems	
3. Parabolic equations in 2-D	6
a. An explicit method	
b. ADI methods	
4. Hyperbolic equations	9
a. Method of characteristics	

   b. CFL condition and Fourier analysis
   c. Upwind scheme and Lax-Wendroff scheme
   d. Finite volume schemes
   e. Conservation properties
5. Special Methods 3
   a. Differentiation matrix and FFT
   b. Smoothness and spectral accuracy
6. Elliptic Equations
   a. Basic iterative schemes
   b. Curved boundaries
   c. Multigrid method
   d. Boundary integral method
   e. Finite element method

Assessment:	Homework	10-30%
	Computer Programs/Project	10-20%
	Quizzes/Tests	20-50%
	Final Exam	30-50%

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