Illinois Institute of Technology

Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, August 22, 2019

4:00–8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.
2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.
3. *DON'T PANIC:* It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.

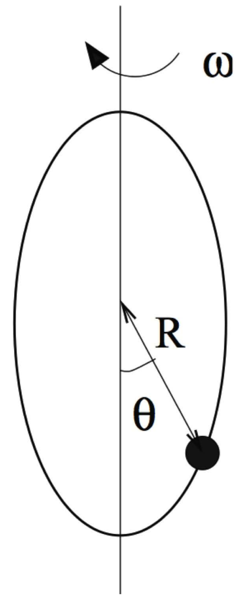
Physical Constants

Speed of light in vacuum	c	=	2.998×10^{8} m/s
Planck's constant	h	=	6.626×10^{-34} J· s
	$\hbar$	=	$h/2\pi$
		=	1.055×10^{-34} J·s
		=	6.582×10^{-16} eV·s
Permeability constant	μ_o	=	$4\pi \times 10^{-7}$ N/A^2
Permittivity constant	$\frac{1}{4\pi\epsilon_o}$	=	8.988×10^{9} N·m^2/C^2
Fine structure constant	α	=	$\frac{e^2}{4\pi\epsilon_o\hbar c}$
		=	$7.30 \times 10^{-3} = \frac{1}{137}$
Gravitational constant	G	=	6.67×10^{-11} m^3/s^2·kg
Avogadro's number	N_A	=	6.023×10^{23} mole^{-1}
Boltzmann's constant	k	=	1.381×10^{-23} J/K
		=	8.617×10^{-5} eV/K
kT at room temperature	k·300 K	=	0.0258 eV
Universal gas constant	R	=	8.314 J/mole·K
Stefan-Boltzmann constant	σ	=	5.67×10^{-8} W/m^2·K^4
Electron charge magnitude	e	=	1.602×10^{-19} C
Electron rest mass	m_e	=	9.109×10^{-31} kg
		=	0.5110 MeV/c^2
Neutron rest mass	m_n	=	1.675×10^{-27} kg
		=	939.6 MeV/c^2
Proton rest mass	m_p	=	1.672×10^{-27} kg
		=	938.3 MeV/c^2
Deuteron rest mass	m_d	=	3.343×10^{-27} kg
		=	1875.6 MeV/c^2
Atomic mass unit (C^{12} = 12)	u	=	1.661×10^{-27} kg
		=	931.5 MeV/c^2
Mass of earth	M_E	=	5.98×10^{24} kg
Radius of earth	R_E	=	6.37×10^{6} m
Mass of sun	M_S	=	1.99×10^{30} kg
Radius of sun	R_S	=	6.96×10^{8} m
Gravitational acceleration at earth's surface	g	=	9.81 m/s^2
Atmospheric pressure		=	1.01×10^{5} N/m^2
Radius of earth's orbit		=	1.50×10^{11} m
Radius of moon's orbit		=	3.84×10^{8} m

Conversion Factors

1 eV	=	1.602×10^{-19} J	1 J	=	6.242×10^{18} eV
1 Å	=	10^{-10} m	1 Fermi	=	10^{-15} m
1 barn (b)	=	10^{-28} m^2	1 in	=	2.54 cm
0° Celsius	=	273.16 K	1 cal	=	4.19 J

Problem 1:



A small bead of mass m slides without friction on a circular hoop of radius R (see figure). The hoop spins with a constant angular velocity ω about its vertical diameter.

(a) Write down the Lagrangian for the bead.

(b) Derive the equation of motion for the bead.

(c) Find all possible equilibrium positions of the bead. Which of them are stable and which are unstable?

(d) Find the frequencies of small oscillations about stable equilibrium positions.

Problem 2: A damped harmonic oscillator consists of a mass m, connected to a vertical spring with spring constant k, immersed in a fluid. The drag force is of the form $-c\mathbf{v}$, where c is a constant and $\mathbf{v}$ is the velocity. Consider small displacements x, from equilibrium.

(a) Show that the displacement

$$x(t) = e^{-\gamma t} A \cos(\omega t + \phi)$$

is a solution of the differential equation of motion for weak damping. Determine ω and γ and the conditions for such solution to exist in terms of the given quantities.

(b) If the frequency of a damped harmonic oscillator is one-half the frequency of the same oscillator with no damping, find the ratio of the maxima of successive oscillations.

Problem 3: A spider is hanging by a silk thread from a tree in Chicago. Find the orientation and the value of the equilibrium angle that the thread makes with the vertical (i.e. with the direction of gravity), taking into account the rotation of the earth.

Assume that the latitude of Chicago is $\theta \approx 42°$ and the radius of the earth is $R \approx 6400$ km.

Problem 4: A satellite travels in a circular orbit of radius r_0. Its rocket motor fires, suddenly increasing its velocity by 8% along its direction of motion. What is the apogee of the new orbit? Make a sketch superimposing the new orbit on the original orbit.

Problem 5: Consider a three-dimensional ideal gas placed into a spherically symmetric potential, given by the formula:

$$V(r) = \begin{cases} \infty, & r \le R \\ U_0 \ln(r/R), & r > R \end{cases}$$

(a) Find a single particle partition function.

A useful Gaussian integral:

$$\int_0^\infty x^2 e^{-x^2/a} dx = \sqrt{\pi a^3}/4$$

(b) Find an N-particle partition function for $N \gg 1$. A useful Stirling formula:

$$N! \approx \left(\frac{N}{e}\right)^N$$

in this case.

(c) What is the highest possible gas temperature?

(d) Find a (Helmholtz) free energy of the gas.

(e) Find an internal energy of the gas.

(f) Find a specific heat (capacity) of the gas.

Problem 6: Calculate a chemical potential for an ideal **two-dimensional** non-relativistic Fermi-gas of a surface density n_S and arbitrary temperature T. A fermion mass is equal to m and spin is 1/2. Then find the chemical potential for a high and low temperature limits.

Problem 7: Using statistical mechanical principles, estimate the mean thickness of the earth's atmosphere in terms of the mass m of a nitrogen molecule, the gravitational acceleration g, and the average atmospheric temperature T. Evaluate your result numerically.

Problem 8:

(a) A cylindrical glass optical fiber (radius R) with index of refraction, n, propagates light by total internal reflection at the air-glass interface. Give an expression for the maximum angle of incidence, α, at the flat end of the fiber.

(b) If a point source of light is used to illuminate the optical fiber, find the distance from the end of the fiber at which the light source must be placed to maximize the intensity of light that comes out of the other end of the fiber.