# Basic Propositional Logic

Academic Resource Center

# In This Presentation…

- Learn how to create and use truth tables
- Learn some basic operations
- Explain the operations in propositions
- Learn about tautologies and how to show them

# Truth Tables

- Useful for determining equivalences and also for showing simple true and false values for variables gone through various operations.
- Two values occupy the spaces in a truth table:
  - “True” value: Either “1” or “T”
  - “False” value: Either “0” or “F”
- In this presentation we will be using the “0” and “1” values

Sample Truth Table (p is our variable and ¬p is an operation)

| **p** | **¬p** |
|---|---|
| 0 | 1 |
| 1 | 0 |

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# Truth Tables

- When creating a truth table, we start off by listing all the variables in one separate column
- Following the variables, we fill in the operations we need to perform to the columns on the left of the variables

- In the case for multiple variables, we list all possible combinations of true and false values for the variables and that will determine the amount of rows we have
  - This number will be $2^n$, where n is the number of variables
  - In the previous slide, the sample truth table that was shown had only 1 variable, p, so n = 1 and the number of rows we expect to have is $2^1 = 2$.

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# Truth Tables

Sample Truth Table:

Variables: p and q → n = 2 and we expect to have $2^2 = 4$ rows

Operation: p ^ q (These operations will be explained in the next few slides)

| p | q | p ^ q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(Note that the first two columns are our variables and the last column is the operation we performed)

# Basic Operations

The NOT operation: ¬

The AND operation: ∧

The OR operation: ∨

The "implication" operation: →

The "equivalence" operation: ↔ or ≡

# NOT Operation

- The NOT operation is quite self-explanatory and is what one would think it to be, negation.
- This operation can only be performed to one variable
- Often denoted by using the symbol “¬” or sometimes by “ ’ ” in other courses
- Example: ¬p or p’

Truth Table:

| p | ¬p or p’ |
|---|---|
| 0 | 1 |
| 1 | 0 |

(Note that the “true” value for ¬p is when p is false)

# NOT Operation

- In plain text, we can describe the operation to be the opposite of what the original value is.

Example: Let p = “cat”

Then ¬p will be “not a cat”

Example: Let p = “tired”

Then ¬p = “not tired”

# AND Operation

- For the AND Operation,  two or more variables are required and will only return “true” if both variables are also “true”.
- Often denoted by the symbol “∧” or “ · ”
- Example: $p \land q$ or $p \cdot q$

Truth Table:

| **p** | **q** | **p ^ q** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

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# AND Operation

- In plain text,

Example: Let p = "tired" and q = "hungry"

Then $p \wedge q$ = "tired and hungry"

Also, $\neg p \wedge q$ = "not tired and hungry"

Or, $p \wedge \neg q$ = "tired and not hungry"

# OR Operation

- For the OR Operation,  two or more variables are required and will return “true” if one or more of the variables are “true”
- Often denoted by the symbol “V” or “+”
- Example: p V q or p + q

Truth Table:

| **p** | **q** | **p V q** |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# OR Operation

In plain text,

Example: Let p = “tired” and q = “hungry”

If “tired and not hungry”, p $\vee$ q is “true”

If “not tired and not hungry”, p $\vee$ q is “false”

# “Implication” Operation

- For implication, it is slightly more complicated and requires two variables. It will return “true” if the initial condition is false, regardless of the value of the second variable, and when both variables are true. (See the truth table below)
- Denoted by the symbol “→”
- Example: p → q

Truth Table:

| p | q | p → q |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# "Implication" Operation

- In plain text, we can use it in the form "If p, then q"

Example: Let p = "hungry" and q = "eat"

$p \rightarrow q$ will be "If I am hungry, I eat."

Example: "You must be 21 to drink."

If p = "John is 21" and q = "John drinks"

Then $q \rightarrow p$ satisfies the statement

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# “Implication” Operation

- $p \rightarrow q$ is also equivalent (the same as) $\neg p \vee q$
- We can verify this using truth tables:

| p | q | ¬p V q |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(Note we receive the same values as the previous table)

Example: Let p = “eat dessert” and q = “eat broccoli”

$p \rightarrow q$ is “You must eat broccoli to eat dessert”

This is the same as $\neg p \vee q$ “Broccoli or no dessert”

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# Example

- Using the operations we have just learned, we will apply them using truth tables:

| p | q | p ^ q | ¬(p ^ q) | ¬p | ¬q | ¬p ^ ¬q | p V q | ¬(p V q) | ¬p ^ ¬q |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

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# Equivalence

- Equivalence requires two variables and will return "true" if both variables have the same value
- Often denoted by the symbol "↔" or "≡"
- Example: $p \leftrightarrow q$ or $p \equiv q$

Truth Table:

| **p** | **q** | **p ↔ q** |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

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# Tautology

- Tautology is very similar to logical equivalence
- When all values are “true” that is a tautology

Example: $p \equiv q$ if and only if $p \leftrightarrow q$ is a tautology

Example: $p \equiv \neg\neg p$ is a tautology

# Tautology

- This can be seen easily through the use of truth tables

Example: Show that p ^ (q V r) ≡ (p ^ q) V (p ^ r) ← statement

| p | q | r | q V r | p ^ (q V r) | p ^ q | p ^ r | (p ^ q) V (p ^ r) | Statement |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

(Note all values are true therefore it is a tautology)

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# Example

Show that (p ^ q) → p is a tautology:

| p | q | p ^ q | (p ^ q) → p |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

All values are true therefore it is a tautology.

# References

- Discrete Mathematics and Its Applications - Rosen 6th Edition